Lorenzo Maccone's research

[Lorenzo Maccone]

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Universita' di Pavia

www.qubit.it

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Main research interests:

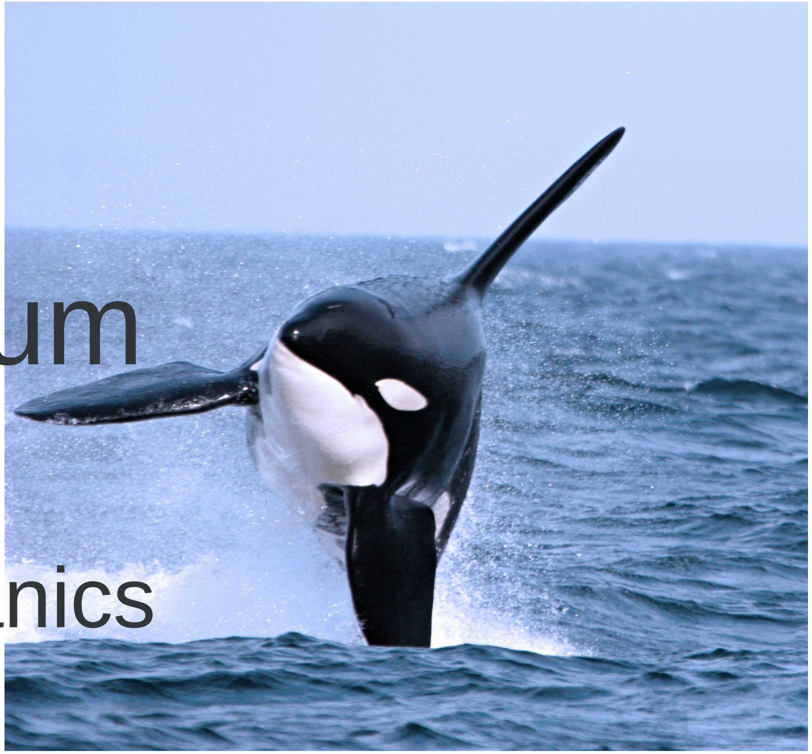
- Quantum technologies

  (use quantum effects for practical applications)

  Quantum metrology
  Quantum information
  Quantum computation
  Quantum cryptography
  Quantum optics

- Foundations of quantum mechanics

  (understand quantum mechanics better).



Quantum metrology?

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Metrology: estimation of a parameter, through measurements.

The estimation is always performed by averaging over N measurements, so that (central limit theorem), the error of the average goes as $1/\sqrt{N}$

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Quantum Metrology: estimation of a parameter with increased precision (thanks to quantum effects, e.g. entanglement)

Usually: $\sqrt{N}$ enhancement:
the error goes as $1/N$

Quantum metrology: main results

1. The **general theory of quantum metrology** (we wrote the paper “quantum metrology”, now it is one of the **pillars** of the quantum technologies flagship):

PRL **96,** 010401 (2006) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2006

Quantum Metrology

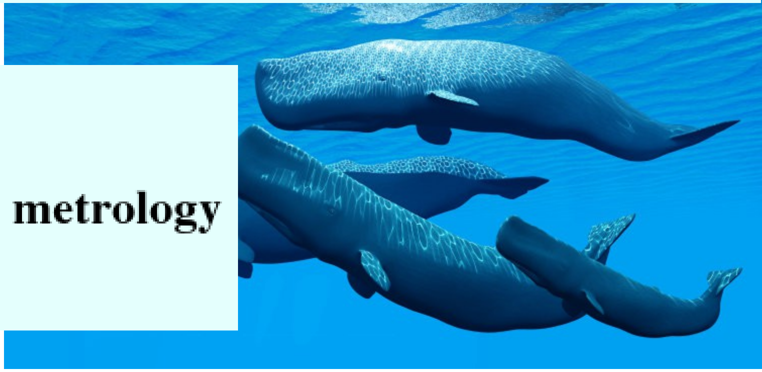
Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

[1]*NEST-CNR-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126, Pisa, Italy*
[2]*MIT, Research Laboratory of Electronics and Department of Mechanical Engineering, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*
[3]*QUIT–Quantum Information Theory Group, Dipartimento di Fisica “A. Volta” Università di Pavia, via Bassi 6, I-27100 Pavia, Italy*

PHYSICAL REVIEW A **88**, 042109 (2013)

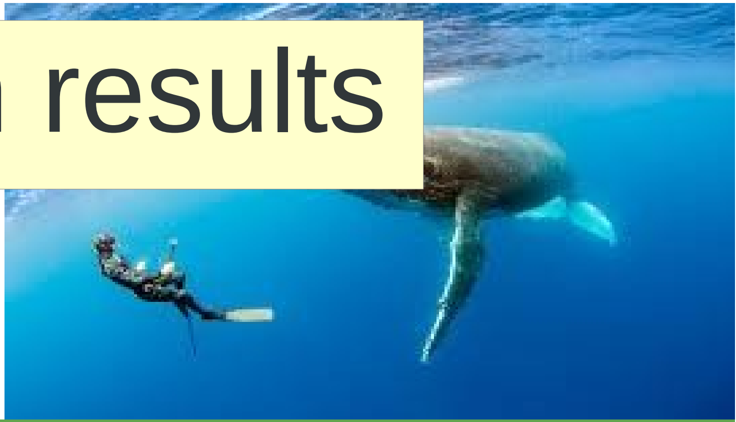
Intuitive reason for the usefulness of entanglement in quantum metrology

Lorenzo Maccone



Quantum metrology: main results

2.A couple of influential reviews.



REVIEW ARTICLES | FOCUS

nature photonics

PUBLISHED ONLINE: 31 MARCH 2011 | DOI: 10.1038/NPHOTON.2011.35

Advances in quantum metrology

Vittorio Giovannetti[1]*, Seth Lloyd[2] and Lorenzo Maccone[3]

Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

Vittorio Giovannetti,[1] Seth Lloyd,[2]* Lorenzo Maccone[3]

1330 19 NOVEMBER 2004 VOL 306 SCIENCE www.sciencemag.org

Quantum metrology: main results

3. Some quantum metrology protocols

Quantum-enhanced positioning and clock synchronization

Vittorio Giovannetti*, Seth Lloyd† & Lorenzo Maccone*

NATURE | VOL 412 | 26 JULY 2001 | www.nature.com 417

VOLUME 87, NUMBER 11 PHYSICAL REVIEW LETTERS 10 SEPTEMBER 2001

Clock Synchronization with Dispersion Cancellation

V. Giovannetti, S. Lloyd,* L. Maccone, and F. N. C. Wong

Quantum metrology: main results

4. Extensions of the theory:

PRL **108,** 260405 (2012) PHYSICAL REVIEW LETTERS week ending 29 JUNE 2012

Quantum Measurement Bounds beyond the Uncertainty Relations

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

PRL **119,** 200502 (2017) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2017

Digital Quantum Estimation

Majid Hassani,[1] Chiara Macchiavello,[2] and Lorenzo Maccone[2]
[1]*Department of Physics, Sharif University of Technology, Tehran 14588, Iran*
[2]*Dipartimento Fisica and INFN Sezione Pavia, University of Pavia, via Bassi 6, I-27100 Pavia, Italy*

Quantum metrology: main results

5. Dealing with noise:

PRL **113,** 250801 (2014) PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2014

Using Entanglement Against Noise in Quantum Metrology

Rafal Demkowicz-Dobrzański[1] and Lorenzo Maccone[2]

PHYSICAL REVIEW A **94**, 012101 (2016)

Usefulness of entanglement-assisted quantum metrology

Zixin Huang,[1] Chiara Macchiavello,[2] and Lorenzo Maccone[2]

Quantum metrology: main results

6. Tomography: many results with the Pavia group!


UNIVERSITÀ DEGLI STUDI DI PAVIA

FACOLTÀ DI SCIENZE MATEMATICHE, FISICHE, NATURALI

DIPARTIMENTO DI FISICA “A. VOLTA”

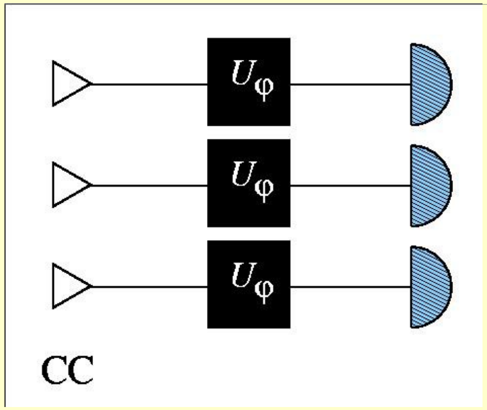
DOTTORATO DI RICERCA IN FISICA - XII CICLO

Quantum tomography:
methods and applications

Main results of QM

Parallel strategies

N probes



Phys. Rev. Lett. **96**, 010401 (2006)

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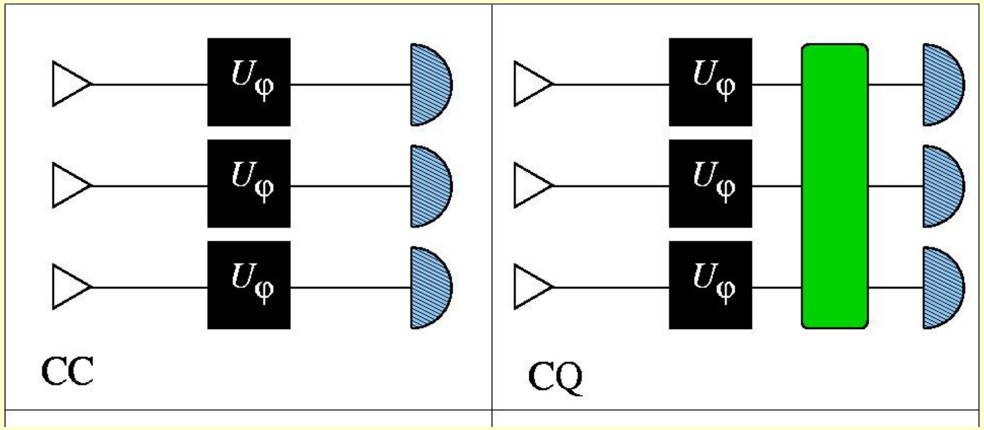

Phys. Rev. Lett. **96**, 010401 (2006)

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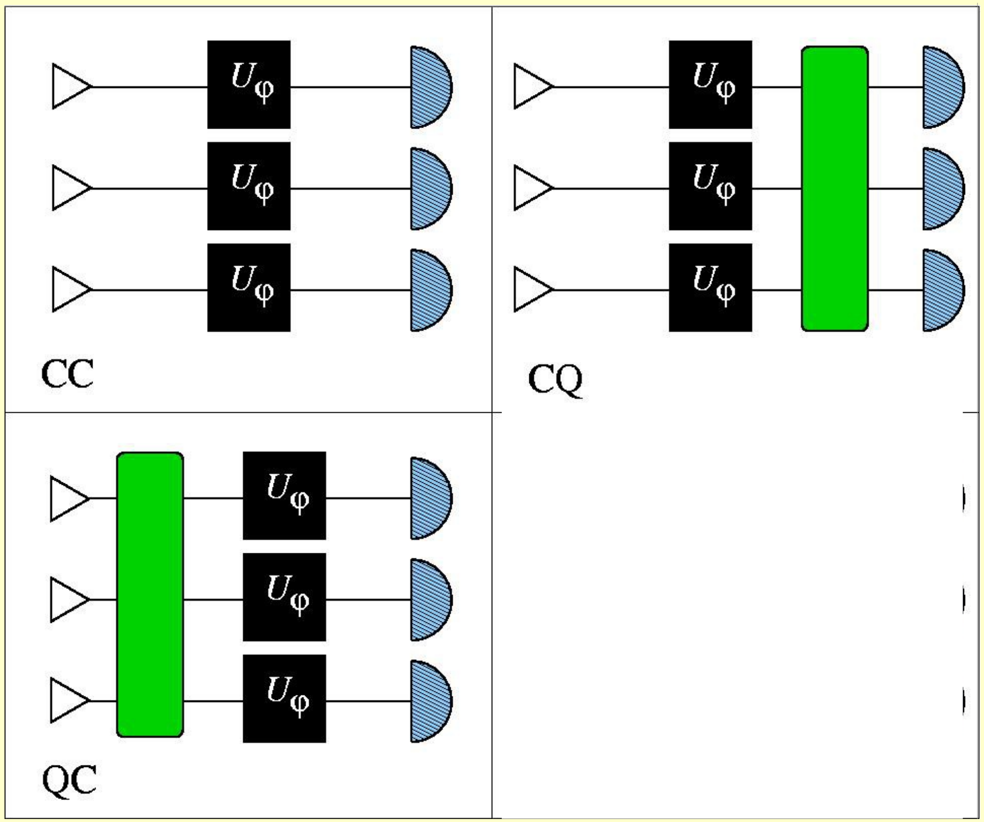

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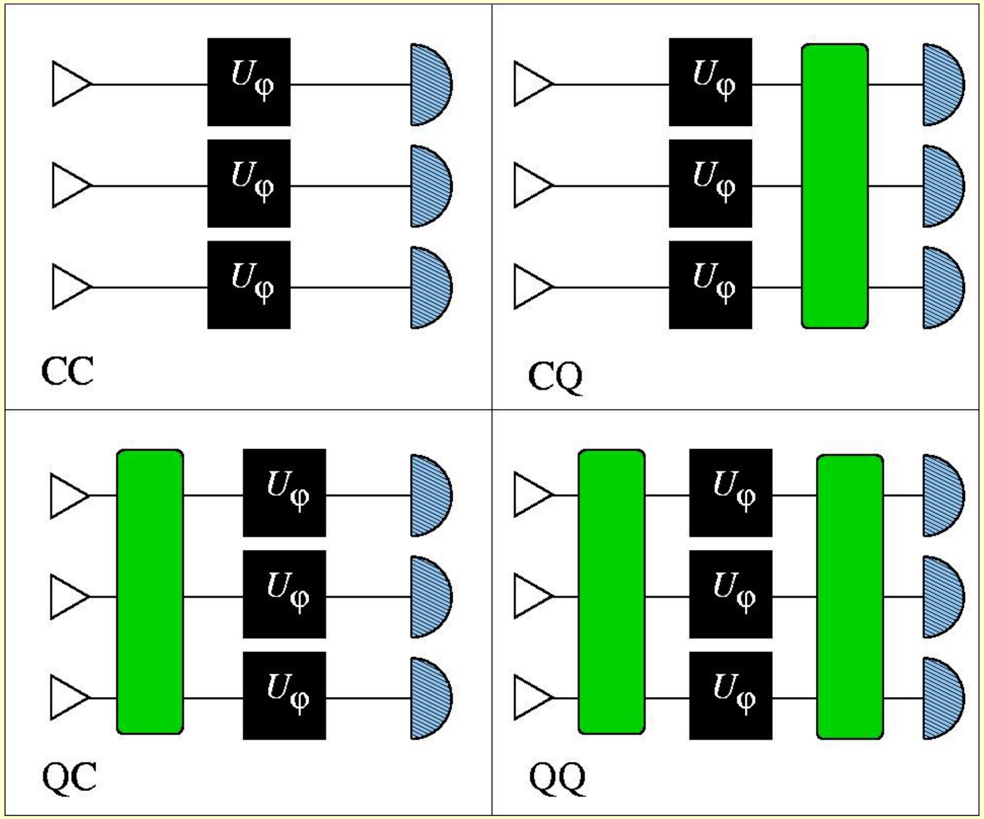

Phys. Rev. Lett. **96**, 010401 (2006)

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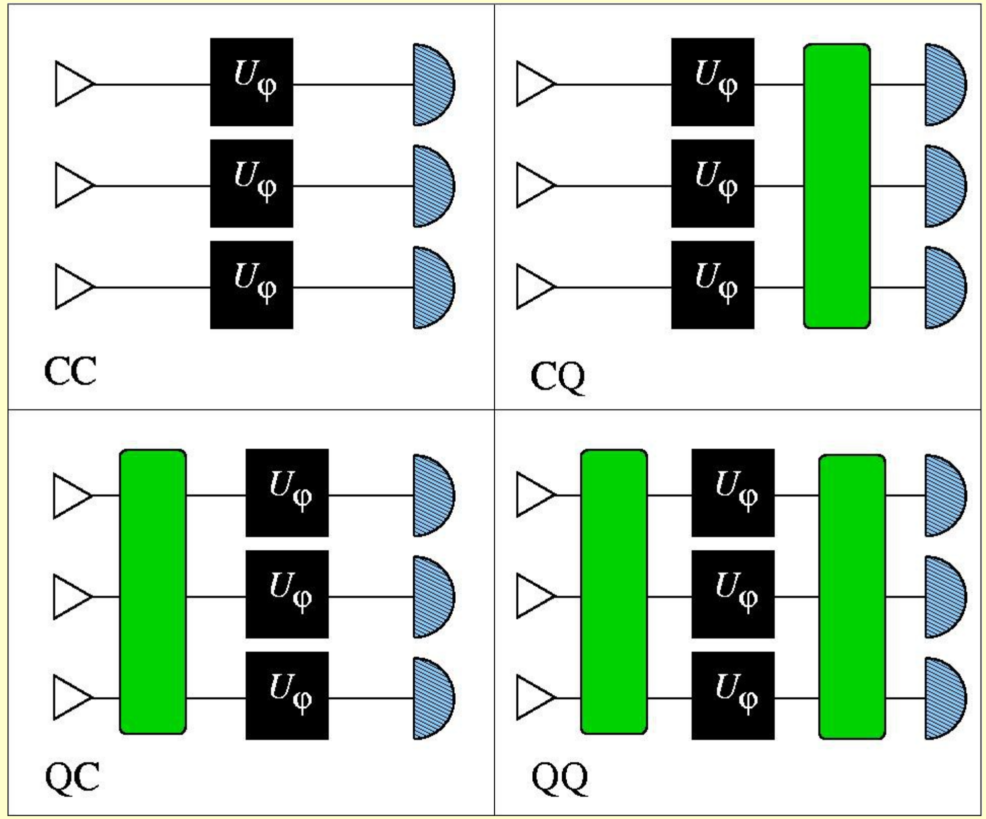
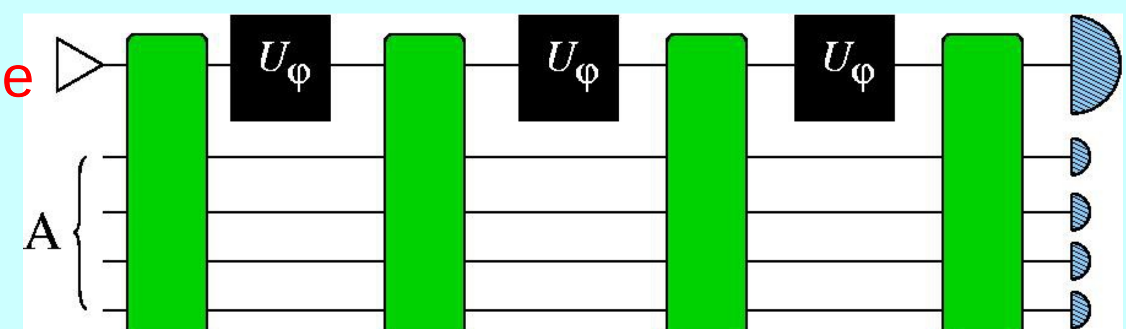


Sequential str.

1 probe



Phys. Rev. Lett. **96**, 010401 (2006)

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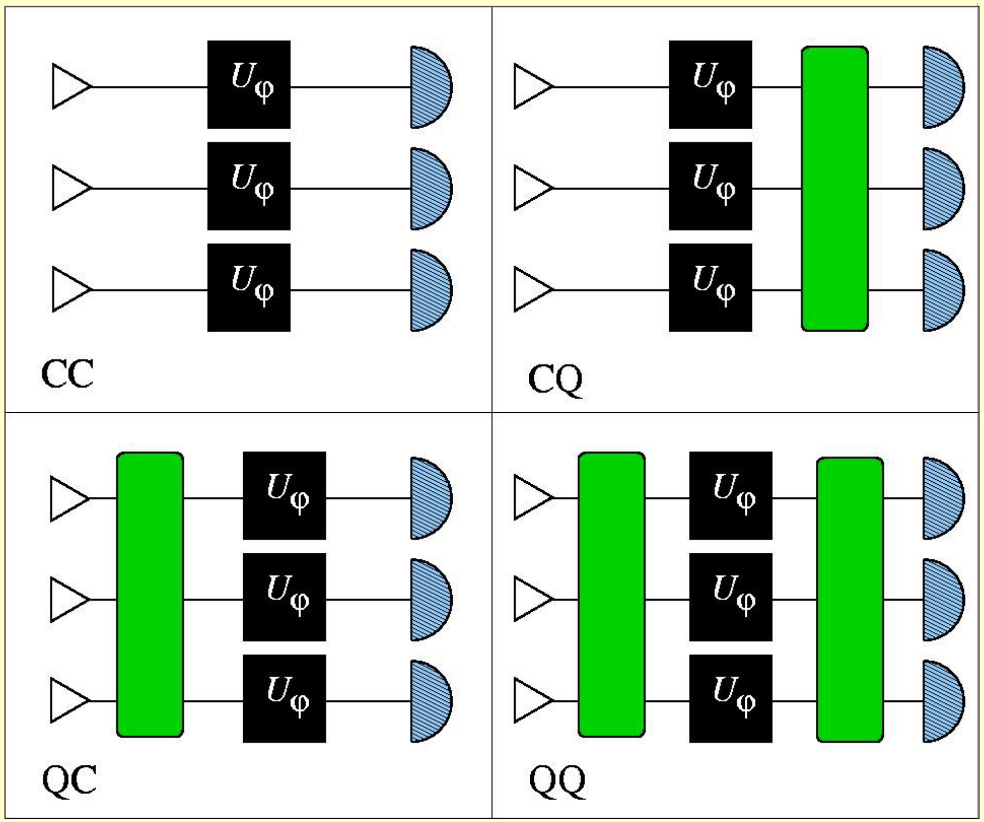

$\rightarrow \dfrac{1}{\sqrt{N}}$

$\rightarrow \dfrac{1}{N}$

Sequential str.

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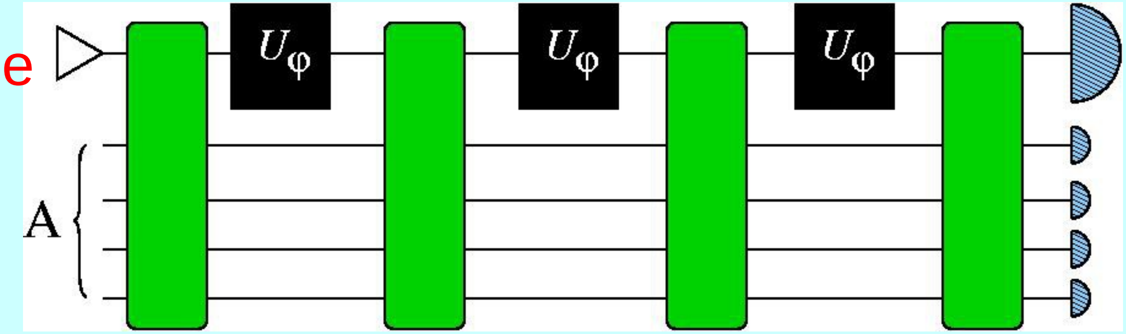

Phys. Rev. Lett. **96**, 010401 (2006)

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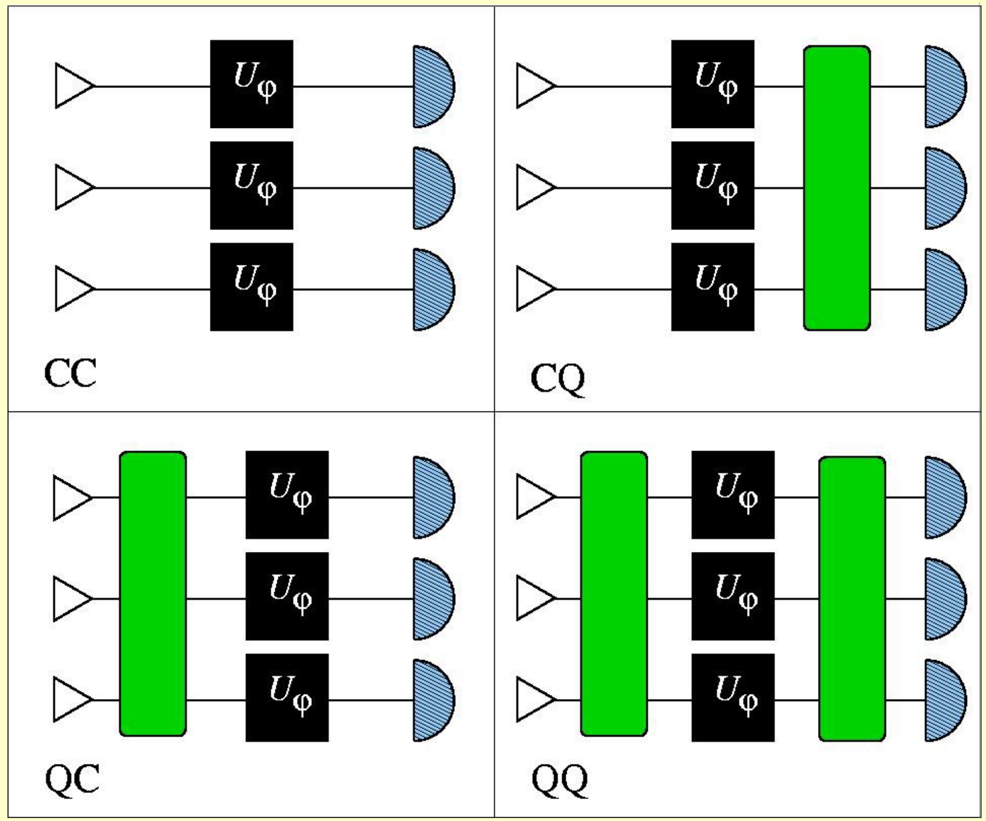

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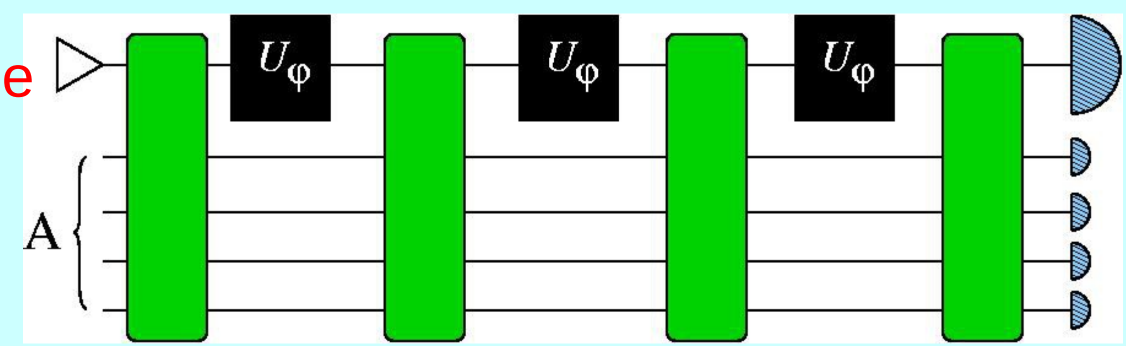


$\rightarrow \frac{1}{N}$

Phys. Rev. Lett. **96**, 010401 (2006)

METRICS

Mutual information case:

Heisenberg bound=Holevo bound: max accessible info

$$I(\vec{m}:\varphi) \leqslant S\Big(\sum_{\varphi} p_{\varphi}\rho_{\varphi}\Big) - \cancel{\sum_{\varphi} p_{\varphi} S(\rho_{\varphi})}$$

prior

encoded probes state

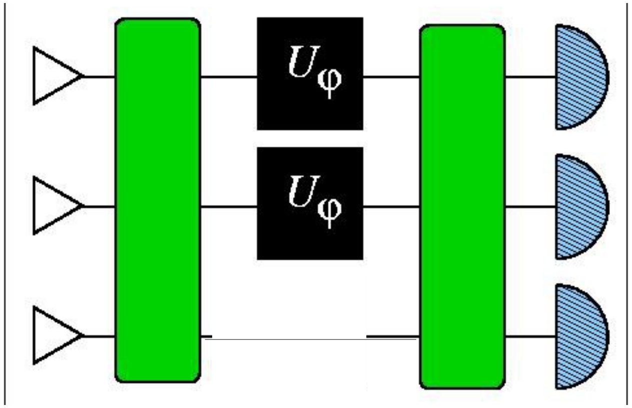
in terms of *N*: $\boxed{I \simeq \log_2 N}$

standard quantum limit: take the square root

$$I \simeq \log_2 \sqrt{N} = \boxed{\tfrac{1}{2}\log_2 N}$$

Ancillary systems

- In the noiseless case **ancillas are useless** to increase precision



- In the **noisy case they may be useful**!

R. Demkowicz-Dobrzanski, L. Maccone, Using entanglement against noise in quantum metrology, PRL 113, 250801.

M. Sbroscia, et al. Experimental ancilla-assisted phase-estimation in a noisy channel, PRA 97, 032305.

Z. Huang, C. Macchiavello, L. Maccone, Noise-dependent optimal strategies for quantum metrology, PRA 97, 032333.

Quantum information: main results

1. Channel capacities:

Fiber optics

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS 16 JANUARY 2004

Classical Capacity of the Lossy Bosonic Channel: The Exact Solution

V. Giovannetti,[1] S. Guha,[1] S. Lloyd,[1,2] L. Maccone,[1] J. H. Shapiro,[1] and H. P. Yuen[3]

ARTICLES
PUBLISHED ONLINE: 11 AUGUST 2013 | DOI: 10.1038/NPHOTON.2013.193
nature photonics

Radio communication

Electromagnetic channel capacity for practical purposes

Vittorio Giovannetti[1]*, Seth Lloyd[2], Lorenzo Maccone[3] and Jeffrey H. Shapiro[4]

Wave guides: PRA 69, 52310

Other channels: PRA 68, 62323 (coauthor: Peter Shor)

Quantum information: main results

2. Quantum entanglement and correlations

PRL **114,** 130401 (2015) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2015

Complementarity and Correlations

Lorenzo Maccone,[1] Dagmar Bruß,[2] and Chiara Macchiavello[1]

3. Multipartite entanglement

PHYSICAL REVIEW A **95**, 042315 (2017)

Multipartite correlations in mutually unbiased bases

David Sauerwein,[1] Chiara Macchiavello,[2] Lorenzo Maccone,[2] and Barbara Kraus[1]

PHYSICAL REVIEW A **97**, 052307 (2018)

Multipartite steering inequalities based on entropic uncertainty relations

Alberto Riccardi, Chiara Macchiavello, and Lorenzo Maccone

Quantum crypto: main results

1. quantum google

PRL **100,** 230502 (2008) PHYSICAL REVIEW LETTERS week ending 13 JUNE 2008

Quantum Private Queries

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

Apparso anche su **Scientific American**, ottobre 2009 e su **Le Scienze**, dicembre 2009 (autore: S. Lloyd).

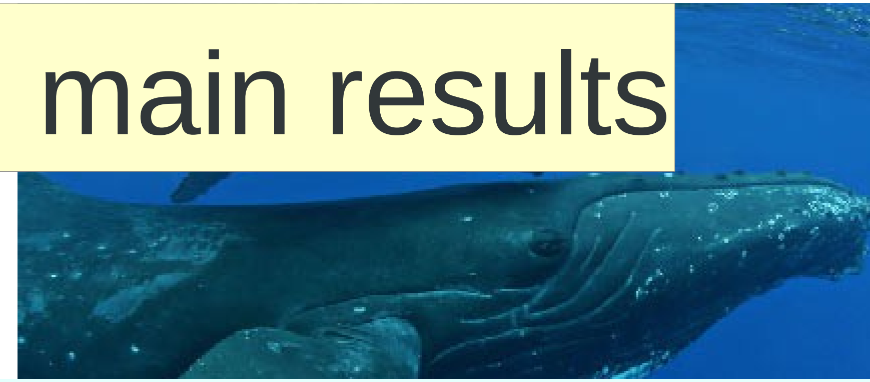
2. blind quantum computation

PRL **111,** 230501 (2013) PHYSICAL REVIEW LETTERS week ending 6 DECEMBER 2013

Efficient Universal Blind Quantum Computation

Vittorio Giovannetti,[1] Lorenzo Maccone,[2] Tomoyuki Morimae,[3,4] and Terry G. Rudolph[3]

Quantum computation: main results



Quantum RAM

PRL **100,** 160501 (2008) PHYSICAL REVIEW LETTERS week ending 25 APRIL 2008

Quantum Random Access Memory

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

(12) **United States Patent**
Lloyd et al.

(10) **Patent No.:** **US 7,764,568 B2**
(45) **Date of Patent:** **Jul. 27, 2010**

(54) **BUCKET BRIGADE ADDRESS DECODING ARCHITECTURE FOR CLASSICAL AND QUANTUM RANDOM ACCESS MEMORIES**

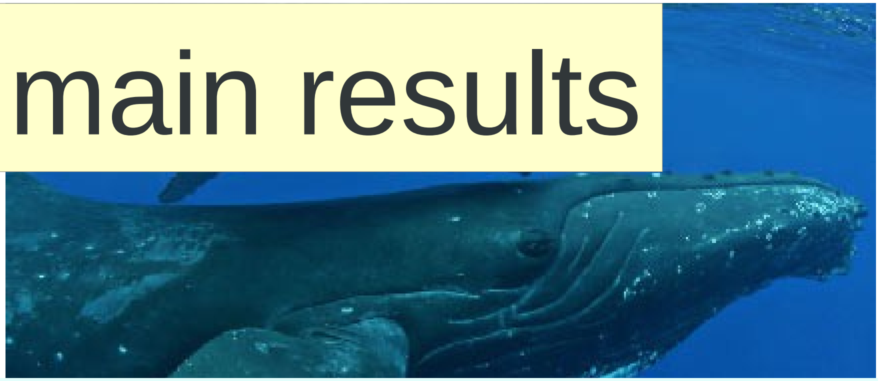
(76) Inventors: **Seth Lloyd**, 18 Weston Rd., Wellesley, MA (US) 02482; **Vittorio Giovannetti**, 4 Via Isola di Montecristo, Pisa (IT) I-56122; **Lorenzo Maccone**, Via Jervis 100, Ivrea (IT) 10015

(56) **References Cited**

U.S. PATENT DOCUMENTS

4,833,677	A	5/1989	Jarwala et al.	
5,784,330	A *	7/1998	Beck et al.	365/230.06
6,529,396	B2 *	3/2003	Hammond	365/63
7,349,288	B1 *	3/2008	Montoye et al.	365/230.06
7,366,031	B2 *	4/2008	Hachmann	365/189.02
2004/0078421	A1	4/2004	Routt	
2005/0059167	A1	3/2005	Vitaliano et al.	

Quantum foundations: main results



1. Uncertainty relations

PRL **113**, 260401 (2014) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2014

Stronger Uncertainty Relations for All Incompatible Observables

Lorenzo Maccone[1] and Arun K. Pati[2,3]

Algebraic Uncertainty Relations

Hubert de Guise,[1] Lorenzo Maccone,[2] Barry C. Sanders,[3,4,*] and Namrata Shukla[3]

Quantum foundations: main results

2. quantum speed limits

EUROPHYSICS LETTERS 1 June 2003

Europhys. Lett., **62** (5), pp. 615–621 (2003)

The role of entanglement in dynamical evolution

V. Giovannetti[1], S. Lloyd[1,2] and L. Maccone[1]

PHYSICAL REVIEW A **67**, 052109 (2003)

Quantum limits to dynamical evolution

Vittorio Giovannetti,[1] Seth Lloyd,[1,2] and Lorenzo Maccone[1]

Quantum foundations: main results

3. quantum TIME

PHYSICAL REVIEW D **92,** 045033 (2015)

Quantum time

Vittorio Giovannetti,[1] Seth Lloyd,[2] and Lorenzo Maccone[3]

Found Phys (2017) 47:1597–1608
DOI 10.1007/s10701-017-0115-2

The Pauli Objection

Juan Leon[1] · Lorenzo Maccone[2]

A fundamental problem in quantizing general relativity

Lorenzo Maccone

Time in quantum mechanics:

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a classical parameter in the Schroedinger eq.

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

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it indicates what is shown
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a classical system!

BUT... **classical systems don't exist**

in a consistent theory of quantum mechanics
(they're just a limiting situation)

define: Time is
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then use a **quantum** system as a clock

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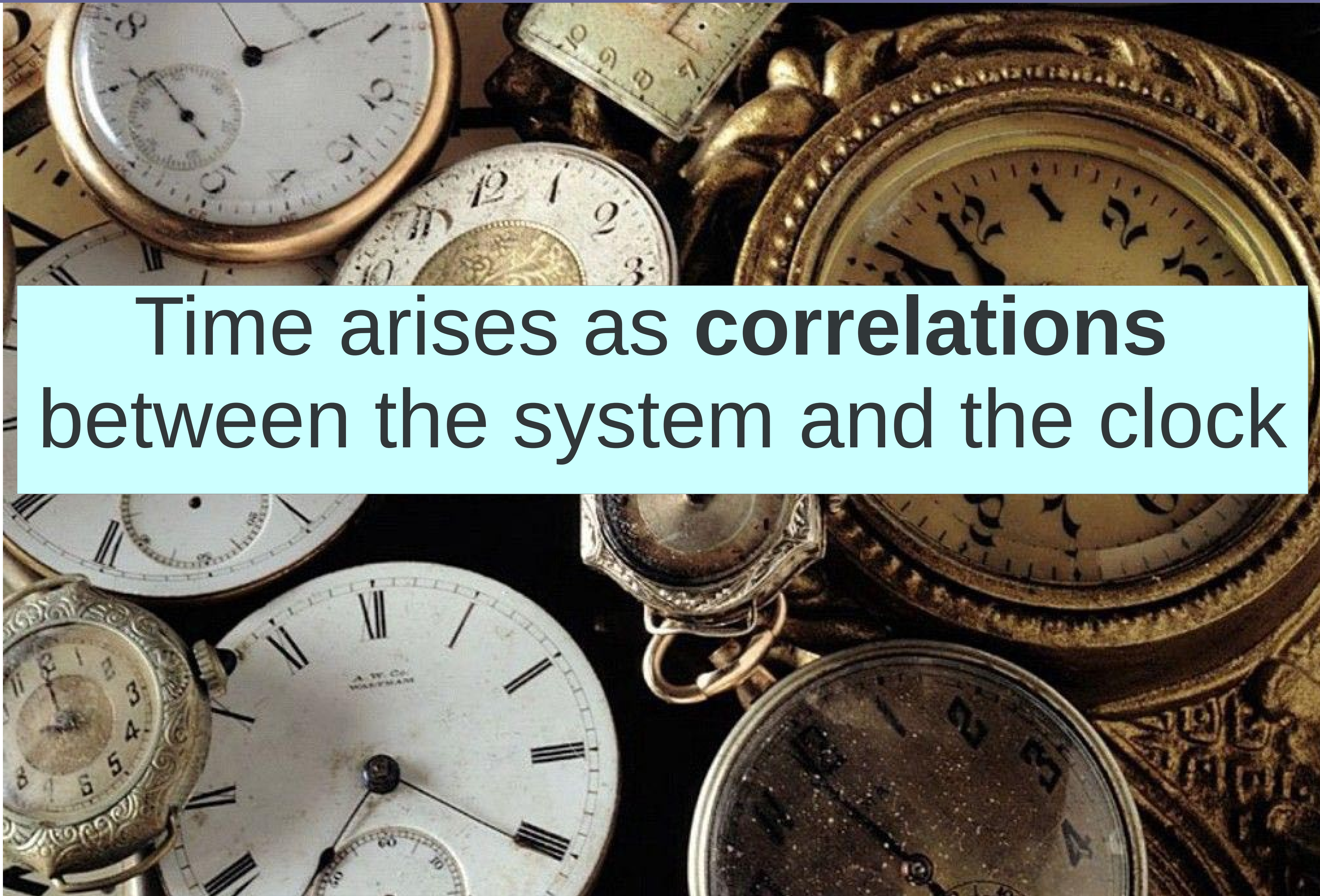
$$\mathcal{H} \equiv \mathcal{L}^2(\mathbb{R})$$

eigenbasis $\{|x\rangle\}$

$|t\rangle$

Time and entanglement



Time arises as **correlations** between the system and the clock

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]
Aharonov and Kaufherr [PRD **30,** 368 (1984)]

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system Hilbert space

constraint operator:

clock “momentum”

system Hamiltonian

$$\hat{\mathbb{J}} := \hbar\hat{\Omega} \otimes \mathbb{1}_S + \mathbb{1}_T \otimes \hat{H}_S \ ,$$

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WdW eq.

bipartite state on $\mathfrak{H} := \mathcal{H}_T \otimes \mathcal{H}_S$

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]

This means that for physical states the Hamiltonian is the generator of time translations

constraint operator:

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How does conventional qm fit in?

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The conventional state: from **conditioning**

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- to the time being *t*: $$|\psi(t)\rangle_S = {}_T\langle t|\Psi\rangle\rangle$$

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$$(\hbar\hat{\Omega}_T + \hat{H}_S)|\Psi\rangle\rangle = 0$$

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"position" representation=Schr eq.

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$${}_T\langle\omega|(\hbar\hat{\Omega}_T + \hat{H}_S)|\Psi\rangle\rangle = 0 \Leftrightarrow \hat{H}_S|\psi(\omega)\rangle_S = -\hbar\omega|\psi(\omega)\rangle_S\ .$$

How does conventional qm fit in?

The conventional state: from **conditioning**

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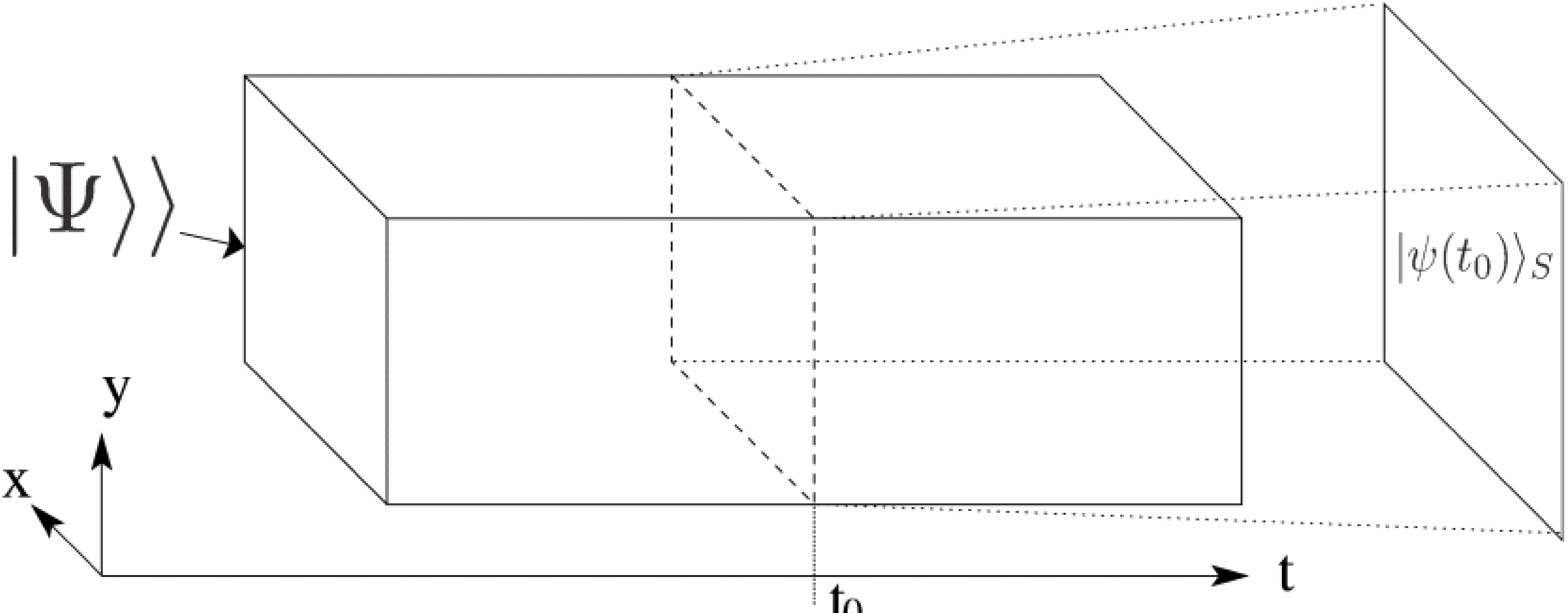
“momentum” representation=time indep. Schr eq.

what I've been saying is that

conventional qm arises in this framework through conditioning.



conditioning?

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All pure solutions to the WdW eq. $\hat{\mathbb{J}}|\Psi\rangle\rangle = 0$

are of the form:

$$|\Psi\rangle\rangle = \int dt\ |t\rangle_T \otimes |\psi(t)\rangle_S$$

conditioning?

All pure solutions to the WdW eq. $\hat{\mathbb{J}}|\Psi\rangle\rangle = 0$

are of the form:

$$|\Psi\rangle\rangle = \int dt \; |t\rangle_T \otimes |\psi(t)\rangle_S$$

$$= \int d\mu(\omega) \; |\omega\rangle_T \otimes |\psi(\omega)\rangle_S \; ,$$

conditioning?

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is a **conditioned state**: the state *given that* the time was *t*

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Other choices are possible!!

Take home message

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My main research interests:
QUANTUM EVERYTHING!!!

www.quantumtechnologies.it/people/maccone