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Ottica non lineare, cenni

13.4 Nonlinear Optics

Generally, the domain of *nonlinear optics* is understood to encompass those phenomena for which electric and magnetic field intensities of higher powers than the first play a dominant role. The Kerr Effect (Section 8.11.3), which is a quadratic variation of refractive index with applied voltage, and thereby electric field, is typical of several long-known nonlinear effects.

The usual classical treatment of the propagation of light—superposition, reflection, refraction, and so forth—assumes a linear relationship between the electromagnetic light field and the responding atomic system constituting the medium. But just as an oscillatory mechanical device (e.g., a weighted spring) can be overdriven into nonlinear response through the application of large enough forces, so too we might anticipate that an extremely intense beam of light could generate appreciable nonlinear optical effects.

As an example of the kinds of fields readily obtainable with the current technology, consider that a good lens can focus a laserbeam down to a spot having a diameter of about 10^{-3} inch or so, which corresponds to an area of roughly 10^{-9} m^2. A 200-megawatt pulse from, say, a *Q*-switched ruby laser would then produce a flux density of 20×10^{16} W/m^2. It follows (Problem 13.27) that the corresponding electric field amplitude is given by

$$E_0 = 27.4\left(\frac{I}{n}\right)^{1/2} \tag{13.30}$$

In this particular case, for $n \approx 1$, the field amplitude is about 1.2×10^8 V/m. This is more than enough to cause the breakdown of air (roughly 3×10^6 V/m) and just several orders of magnitude less than the typical fields holding a crystal together, the latter being roughly about the same as the cohesive field on the electron in a hydrogen atom (5×10^{11} V/m). [redacted] l-

As you may recall, the electromagnetic field of a lightwave propagating through a medium exerts forces on the loosely bound outer or valence electrons. Ordinarily, these forces are quite small, and in a linear isotropic medium the resulting electric polarization is parallel with and directly proportional to the applied field. In effect, the polarization follows the field; if the latter is harmonic, the former will be harmonic as well. Consequently, one can write

$$P = \epsilon_0 \chi E \tag{13.31}$$

where χ is a dimensionless constant known as the electric susceptibility, and a plot of P versus E is a straight line. Quite obviously in the extreme case of very high fields, we can expect that P will become saturated; in other words, it simply cannot increase linearly indefinitely with E (just as in the familiar case of ferromagnetic materials, where the magnetic moment becomes saturated at fairly low values of H). Thus we can anticipate a gradual increase of the ever-present, but usually insignificant, nonlinearity as E increases. Since the directions of $\vec{\mathbf{P}}$ and $\vec{\mathbf{E}}$ coincide in the simplest case of an isotropic

medium, we can express the polarization more effectively as a series expansion:

$$P = \epsilon_0(\chi E + \chi_2 E^2 + \chi_3 E^3 + \cdots) \qquad (13.32)$$

The usual linear susceptibility, χ, is much greater than the coefficients of the nonlinear terms χ_2, χ_3, and so on, and hence the latter contribute noticeably only at high-amplitude fields. Now suppose that a lightwave of the form

$$E = E_0 \sin \omega t$$

is incident on the medium. The resulting electric polarization

$$P = \epsilon_0 \chi E_0 \sin \omega t + \epsilon_0 \chi_2 E_0^2 \sin^2 \omega t + \epsilon_0 \chi_3 E_0^3 \sin^3 \omega t + \cdots \tag{13.33}$$

can be rewritten as

$$P = \epsilon_0 \chi E_0 \sin \omega t + \frac{\epsilon_0 \chi_2}{2} E_0^2 (1 - \cos 2\omega t) + \frac{\epsilon_0 \chi_3}{4} E_0^3 (3 \sin \omega t - \sin 3\omega t) + \cdots \quad (13.34)$$

As the harmonic lightwave sweeps through the medium, it creates what might be thought of as a polarization wave, that is, an undulating redistribution of charge within the material in response to the field. If only the linear term were effective, the electric polarization wave would correspond to an oscillatory current following along with the incident light. The light thereafter reradiated in such a process would be the usual refracted wave generally propagating with a reduced speed v and having the same frequency as the incident light. In contrast, …………→>>

higher-order terms in Eq. (13.33) implies that the polarization wave does not have the same harmonic profile as the incident field. In fact, Eq. (13.34) can be likened to a Fourier series representation of the distorted profile of $P(t)$.

13.4.1 Optical Rectification

The second term in Eq. (13.34) has two components of great interest. First there is a *dc* or *constant bias polarization* varying as E_0^2. Consequently, if an intense plane-polarized beam traverses an appropriate (piezoelectric) crystal, the presence of the quadratic nonlinearity will, in part, be manifest by a constant electric polarization of the medium. A voltage difference, proportional to the beam's flux density, will accordingly appear across the crystal. This effect, in analogy to its radiofrequency counterpart, is known as optical rectification.

13.4.2 Harmonic Generation

The cos $2\omega t$ term [Eq. (13.34)] corresponds to a variation in electric polarization at twice the fundamental frequency (i.e., at twice that of the incident wave). The reradiated light that arises from the driven oscillators also has a component at this same frequency, 2ω, and the process is spoken of as **second-harmonic generation**, or SHG for short.

Peter A. Franken and several coworkers at the University of Michigan in 1961 were the first to observe SHG experimentally. They focused a 3-kW pulse of red (694.3 nm) ruby laserlight onto a quartz crystal. Just about one part in 10^8 of this incident wave was converted to the 347.15-nm ultraviolet second harmonic.

Notice that, for a given material, if *P(E)* is an odd function, that is, if reversing the direction of the $\vec{\mathbf{E}}$-field simply reverses the direction of $\vec{\mathbf{P}}$, the even powers of E in Eq. (13.32) must vanish. But this is just what happens in an isotropic medium, such as glass or water—there are no special directions in a liquid. Moreover, in crystals like calcite, which are so structured as to have what's known as a *center of symmetry* or an *inversion center*,

tered. Thus no even harmonics can be produced by materials of this sort. Third-harmonic generation (THG), however, can exist and has been observed in several materials (p. 598) including calcite.

The requirement for SHG that a crystal not have inversion symmetry is also necessary for it to be piezo-electric.

quartz, potassium dihydrogen phosphate (KDP), or ammonium dihydrogen phosphate (ADP)] undergoes an asymmetric distortion of its charge distribution, thus producing a voltage. Of the 32 crystal classes, 20 are of this kind and may therefore be useful in SHG. The simple scalar expression [Eq. (13.32)] is actually not an adequate description of a typical dielectric crystal. Things are a good deal more complicated,

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A major difficulty in generating copious amounts of second-harmonic light arises from the frequency dependence of the refractive index, that is, dispersion. At some initial point where the incident or ω-wave, generates the second-harmonic or 2ω-wave, the two are coherent. As the ω-wave propagates through the crystal, it continues to generate additional contributions of second-harmonic light, which all combine totally constructively only if they maintain a proper phase relationship.

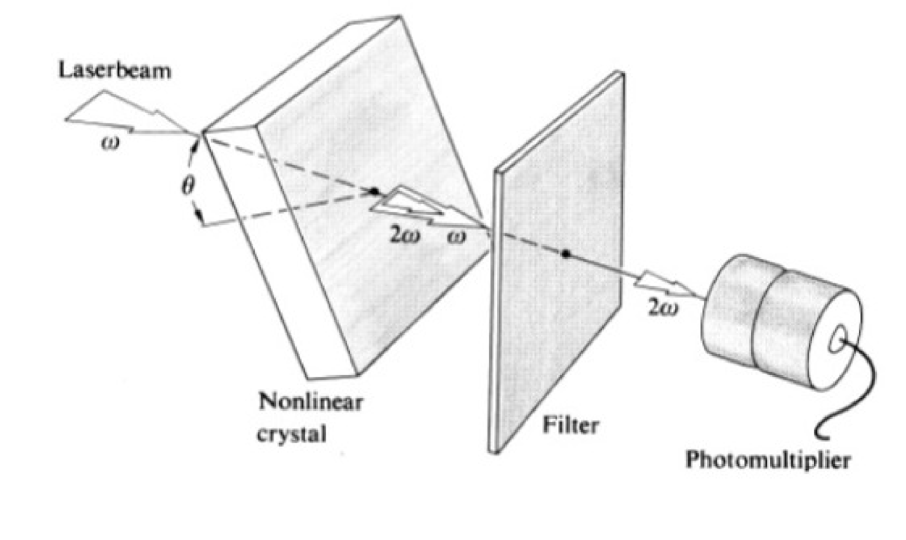
wave. Thus the newly emitted second harmonic periodically falls out-of-phase with some of the previously generated 2ω-waves. When the irradiance of the second harmonic, $I_{2\omega}$, emerging from a plate of thickness ℓ is computed,† it turns out to be

$$I_{2\omega} \propto \frac{\sin^2 [2\pi(n_\omega - n_{2\omega})\ell/\lambda_0]}{(n_\omega - n_{2\omega})^2} \tag{13.35}$$

(see Fig. 13.56). This yields the result that $I_{2\omega}$ has its maximum value when $\ell = \ell_c$, where

$$\ell_c = \frac{1}{4} \frac{\lambda_0}{|n_\omega - n_{2\omega}|} \tag{13.36}$$

This is commonly known as the *coherence length* (although a different name would be better), and it's usually of the order of only about $20\lambda_0$. Despite this, efficient SHG can be accomplished by a procedure known as *index matching*, which



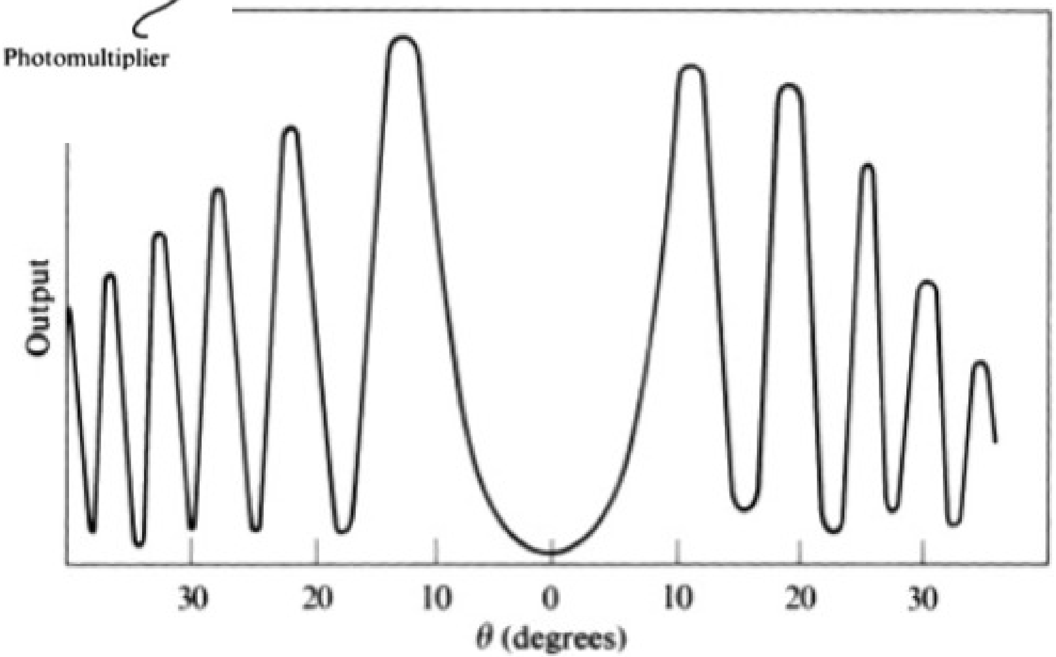
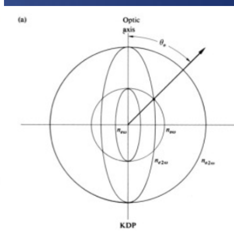
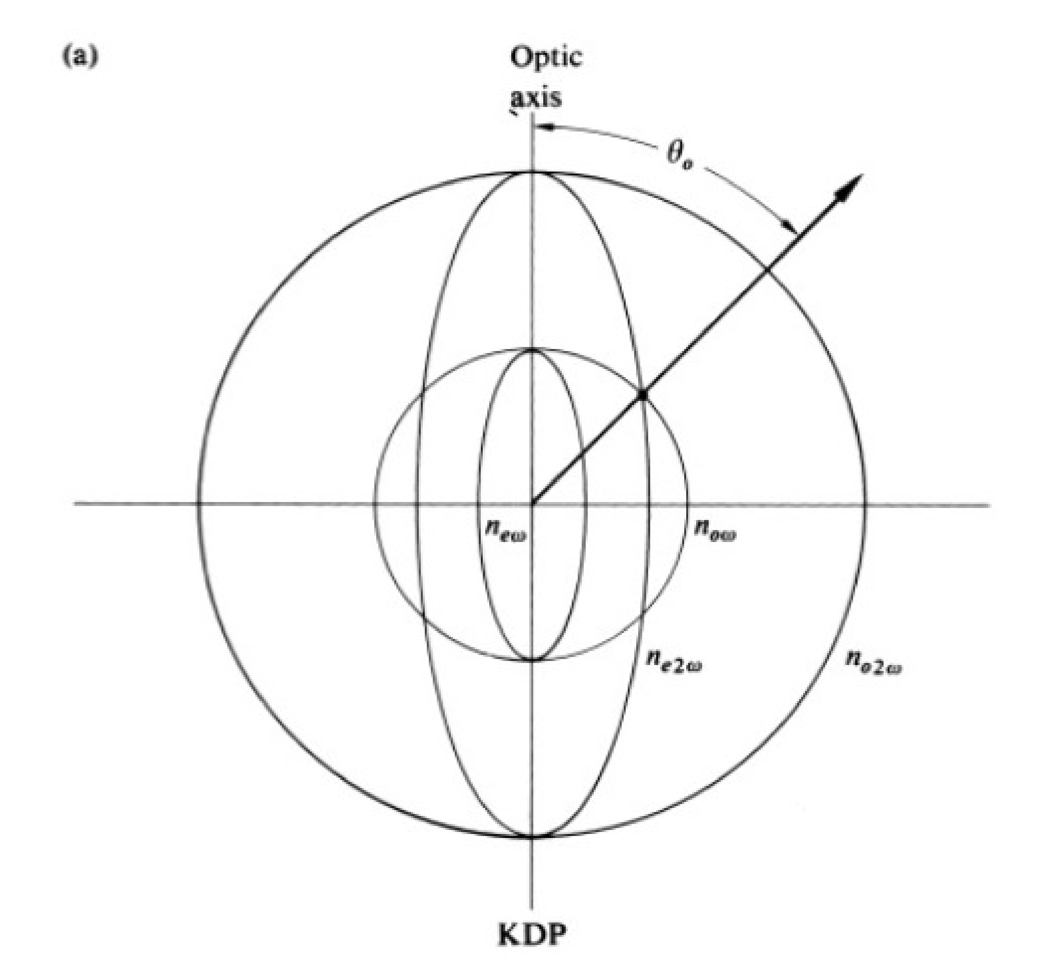


Figure 13.56 Second-harmonic generation as a function of θ for a 0.78-mm thick quartz plate. Peaks occur when the effective thickness is an even multiple of ℓ_c. [From P. D. Maker, R. W. Terhune, M. Nisenoff, and C. M. Savage, *Phys. Rev. Letters* **8**, 21 (1962).]

arranges things so that $n_\omega = n_{2\omega}$. A commonly used SHG material is KDP. It is piezoelectric, transparent, and also negatively uniaxially birefringent. Furthermore, it has the interesting property that if the fundamental light is a linear polarized *ordinary wave*, the resulting second harmonic will be an *extraordinary wave*. As can be seen from Fig. 13.57, if light propagates within a KDP crystal at the specific angle θ_0 with



respect to the optic axis, the index, $n_{0\omega}$, of the ordinary fundamental wave will precisely equal the index of the extraordinary second harmonic $n_{e2\omega}$. The second-harmonic wavelets will then interfere constructively, thereupon increasing the conversion efficiency by several orders of magnitude. Secrespect to the optic axis, the index, $n_{0\omega}$, of the ordinary fundamental wave will precisely equal the index of the extraordinary second harmonic $n_{e2\omega}$. The second-harmonic wavelets will then interfere constructively, thereupon increasing the conversion efficiency by several orders of magnitude. Sec-



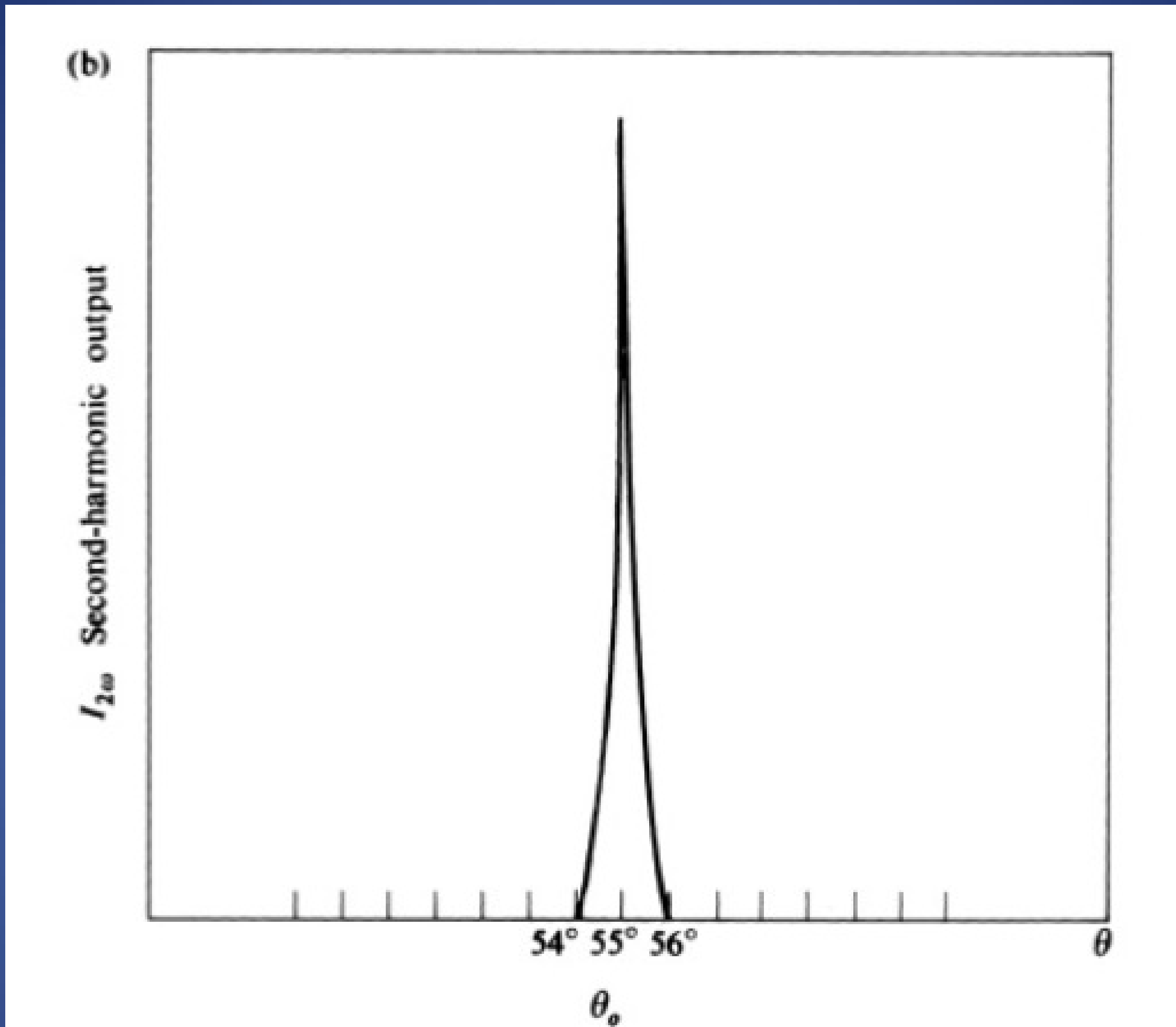


Figure 13.57 Refractive index surface for KDP. (*b*) $I_{2\omega}$ versus crystal orientation in KDP. (From Maker et al.)

13.4.3 Frequency Mixing

Another situation of considerable practical interest involves the *mixing* of two or more primary beams of different frequencies within a nonlinear dielectric. The process can most easily be appreciated by substituting a wave of the form

$$E = E_{01} \sin \omega_1 t + E_{02} \sin \omega_2 t \qquad (13.37)$$

into the simplest expression for P given by Eq. (13.32). The second-order contribution is then

$$\epsilon_0 \chi_2 (E_{01}^2 \sin^2 \omega_1 t + E_{02}^2 \sin^2 \omega_2 t + 2E_{01}E_{02} \sin \omega_1 t \sin \omega_2 t)$$

The first two terms can be expressed as functions of $2\omega_1$ and $2\omega_2$, respectively, while the last quantity gives rise to sum and difference terms, $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$.

Various oscillator configurations have since evolved, with other nonlinear materials used as well, such as barium sodium niobate. The optical parametric oscillator is a laser-like, broadly tunable source of coherent radiant energy in the IR to the UV.

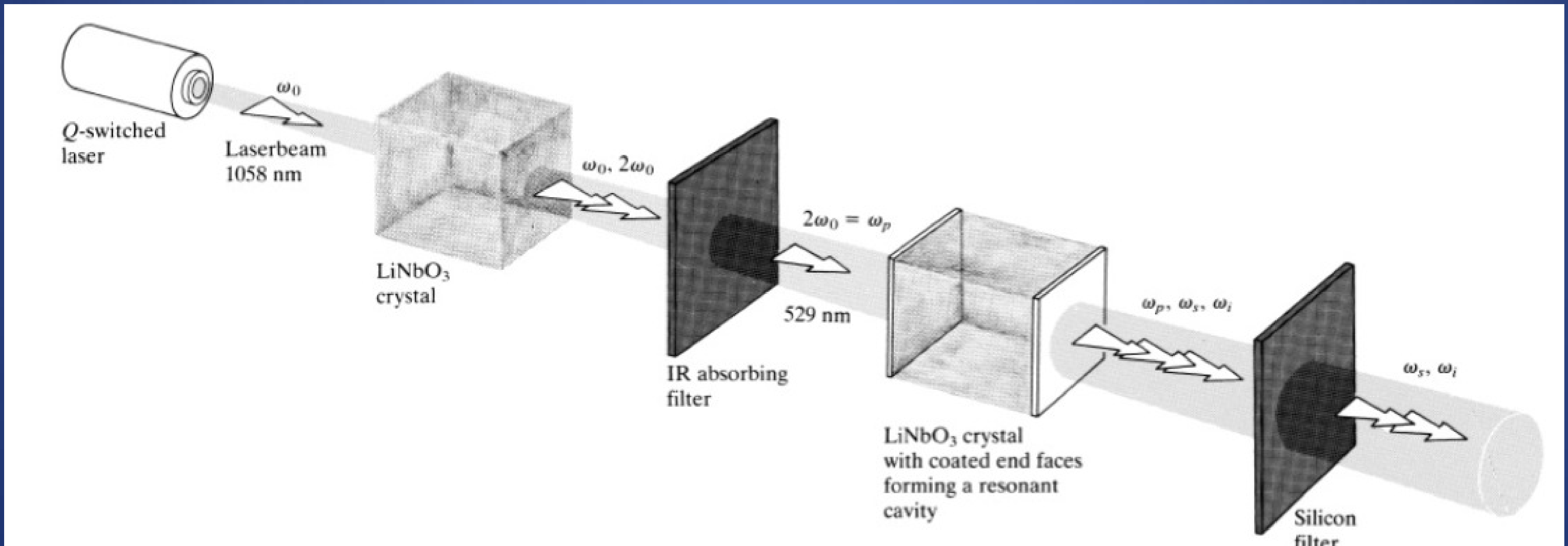


Figure 13.58 An optical parametric oscillator.

[After J. A. Giordmaine and R. C. Miller, *Phys. Rev. Letters* **4**, 973 (1965).]

13.4.4 Self-Focusing of Light

When a dielectric is subjected to an electric field that varies in space, in other words, when there is a gradient of the field parallel to $\vec{\mathbf{P}}$, an internal force will result. This has the effect of altering the density, changing the permittivity, and thereby varying the refractive index, and this in both linear and nonlinear isotropic media. Suppose then that we shine an intense laserbeam with a transverse Gaussian flux-density distribution onto a specimen. The induced refractive-index variations will cause the medium in the region of the beam to function much as if it were a positive lens. Accordingly, the beam contracts, the flux density increases even more, and the contraction continues in a process known as self-focusing. The effect can be sustained until the beam reaches a limiting filament diameter (of about 5×10^{-6} m), being totally internally reflected as if it were in a fiberoptic element embedded within the medium.*

Da confrontare con divergenza λ dipendente e anti diffrazione

Riferimenti:

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Anti-diffraction of light

Using an electro-optic effect, submicrometre-sized beams have been shown to exhibit non-paraxial propagation over 1,000 Rayleigh lengths. The discovery does not require inhomogeneous or lossy media like plasmon waveguiding.