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Gaussian Beams

Plane waves, though important, are not the only solutions to Maxwell's Equations. As we saw in the previous chapter, the differential wave equation allows many solutions, among which are cylindrical and spherical waves (Fig. 3.15).

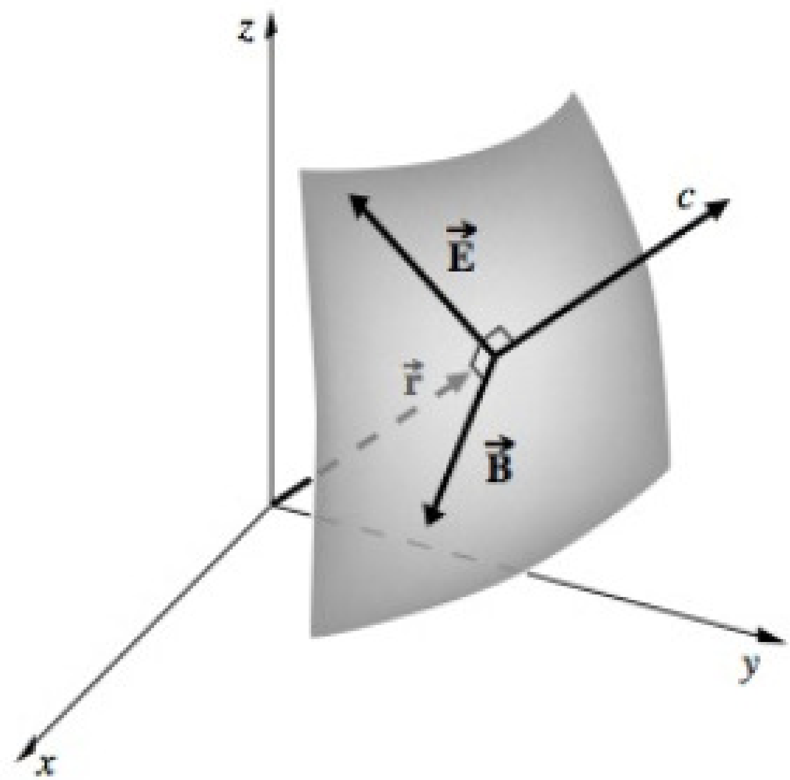
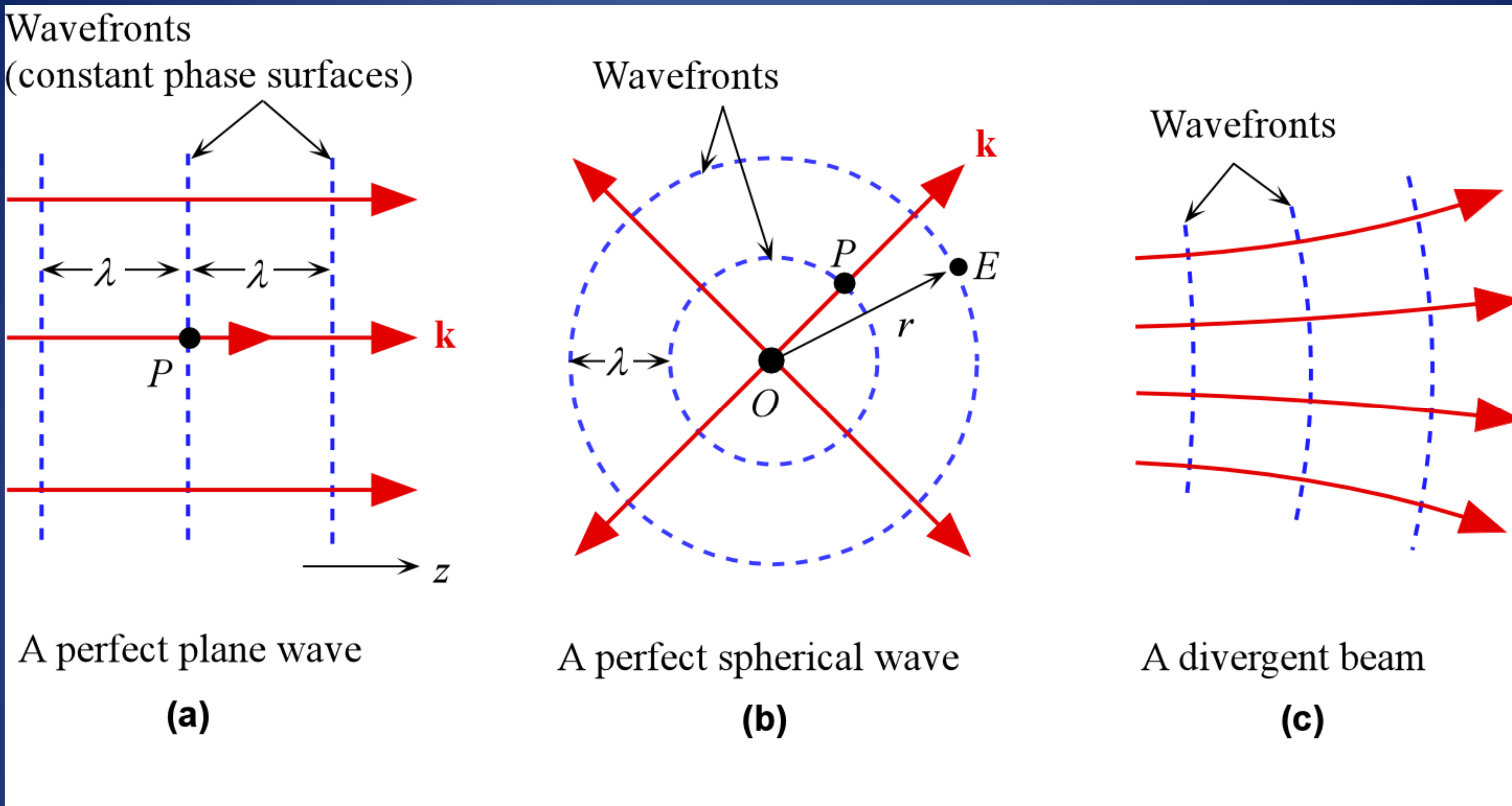


Figure 3.15 Portion of a spherical wavefront far from the source.

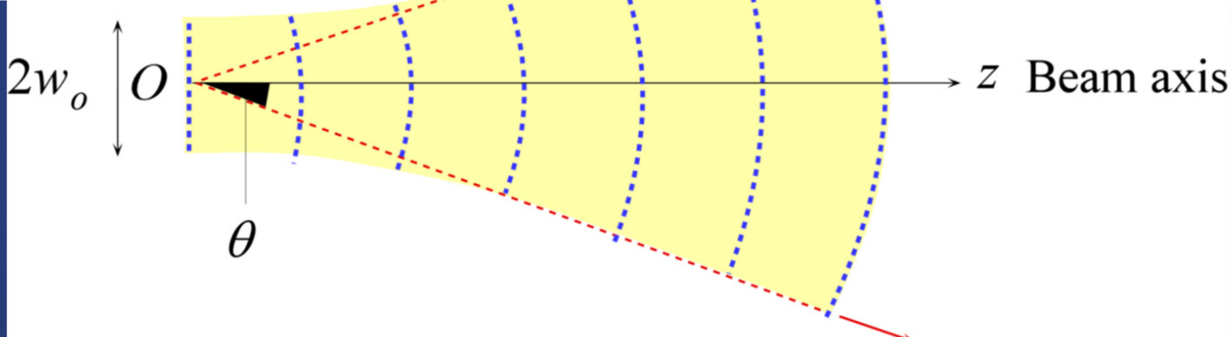


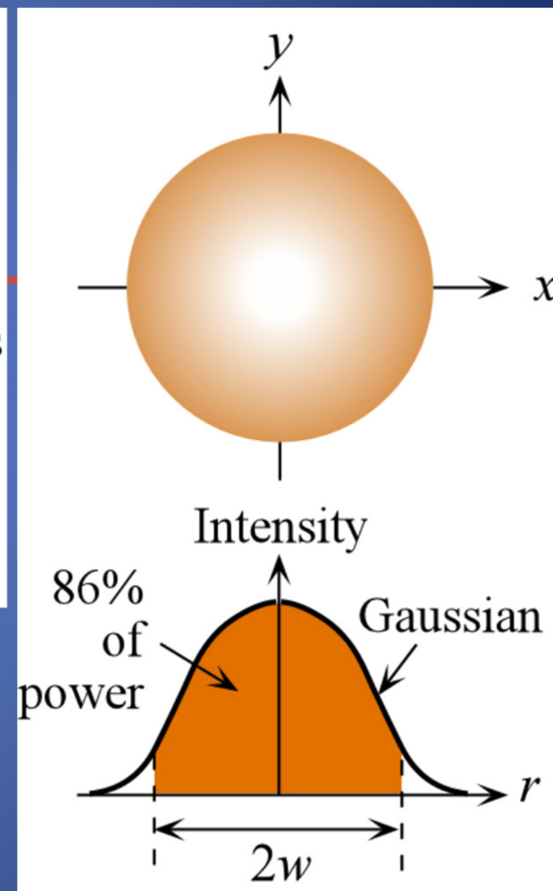
Examples of possible EM waves. (a) A perfect plane wave. (b) A perfect spherical wave. (c) A divergent beam.

Optical divergence refers to the angular separation of wave vectors on a given wavefront.

beam divergence.

$$2\theta = \frac{4\lambda}{\pi(2w_o)}$$

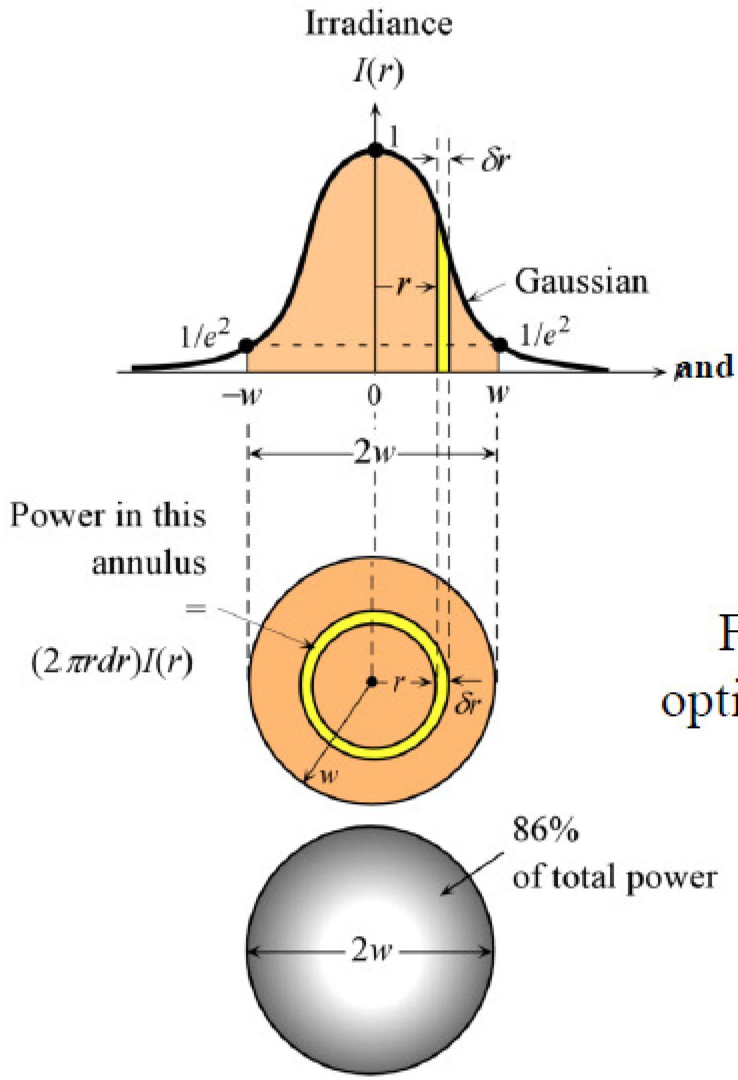




Gaussian Beams

Wavefronts of a Gaussian light beam.

The radiation emitted from a laser can be approximated by a Gaussian beam.



Power in a Gaussian Beam

$$I(r) = I(0)\exp[-2(r/w)^2]$$

Area of a circular thin strip (annulus) with radius r is $2\pi r dr$. Power passing through this strip is proportional to $I(r)(2\pi r)dr$

$$\text{Fraction of optical power within } 2w = \frac{\int_0^w I(r)2\pi r dr}{\int_0^\infty I(r)2\pi r dr} = 0.865$$

con

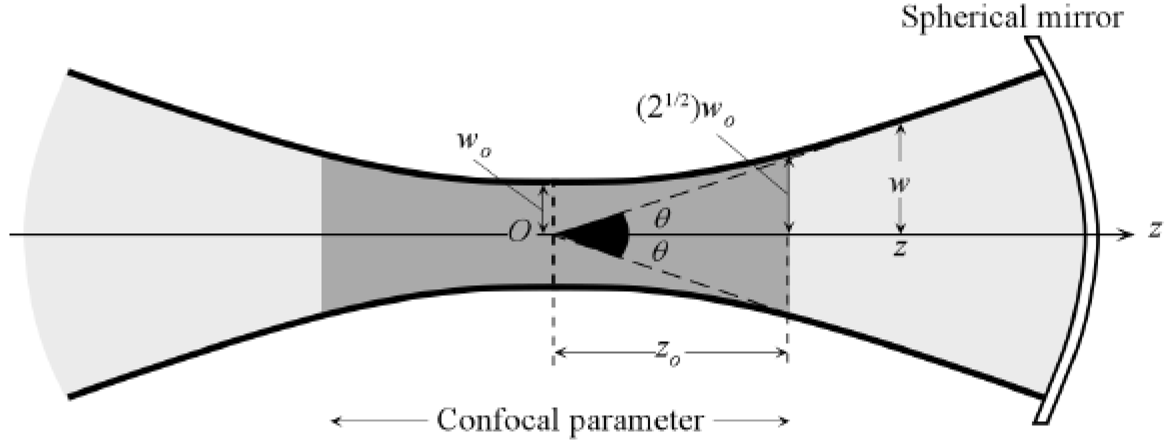
$E(w) = E_0/e$

$I(w) = I_0/e^2$

E_0 e I_0 sono i corrispondenti valori “assiali” (per $z = 0$) di campo elettrico e radianza

Una più completa analisi delle onde EM in cavità porta a

Gaussian Beams



Rayleigh range

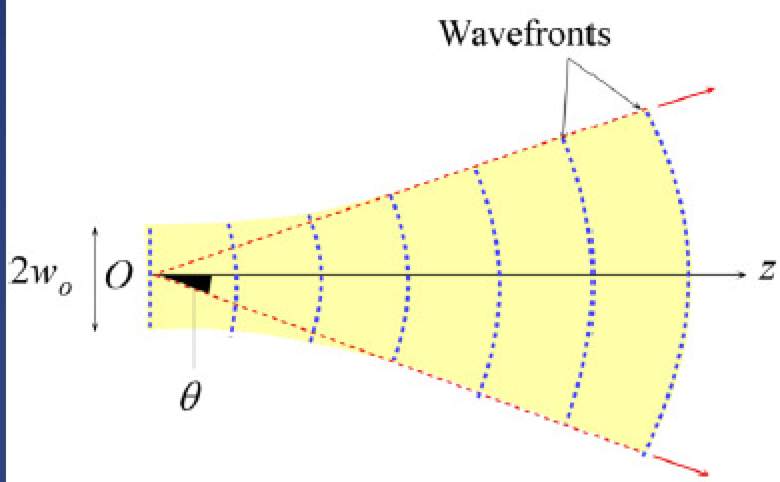
$$z_o = \frac{\pi w_o^2}{\lambda}$$

Con z_0 valore di z per cui $w = \sqrt{2}\, w_0$

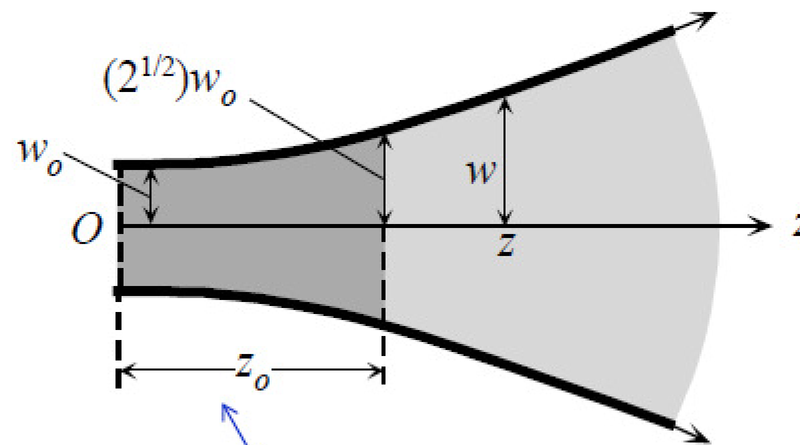
$$2w = 2w_o\left[1+\left(\frac{z}{z_o}\right)^2\right]^{1/2}$$

$$2w = 2w_o\left[1+\left(\frac{z\lambda}{\pi w_o}\right)^2\right]^{1/2}$$

Gaussian Beams



$2\theta_o$ = Far field divergence



Rayleigh range

$$z_o = \pi w_o^2/\lambda$$

EXAMPLE 1.1.1 A diverging laser beam

Consider a He-Ne laser beam at 633 nm with a spot size of 1 mm. Assuming a Gaussian beam, what is the divergence of the beam? What are the Rayleigh range and the beam width at 25 m?

Solution

Using Eq. (1.1.7), we find

$$2\theta = \frac{4\lambda}{\pi(2w_o)} = \frac{4(633 \times 10^{-9}\text{ m})}{\pi(1 \times 10^{-3}\text{ m})} = 8.06 \times 10^{-4}\text{ rad} = 0.046°$$

The Rayleigh range is

$$z_o = \frac{\pi w_o^2}{\lambda} = \frac{\pi\left[(1 \times 10^{-3}\text{ m})/2\right]^2}{(633 \times 10^{-9}\text{ m})} = 1.24\text{ m}$$

The beam width at a distance of 25 m is

$$2w = 2w_o\left[1 + (z/z_o)^2\right]^{1/2} = (1 \times 10^{-3}\text{ m})\left\{1 + \left[(25\text{ m})/(1.24\text{ m})\right]^2\right\}^{1/2}$$

$$= 0.0202\text{ m} \quad \text{or} \quad 20\text{ mm.}$$

3.3.2 Irradiance

When we talk about the "amount" of light illuminating a surface, we are referring to something called the **irradiance**,* denoted by I—the ***average energy per unit area per unit time***.

radianza ed
intensità di un'onda em

*In the past physicists generally used the word *intensity* to mean the flow of *energy per unit area per unit time*. By international, if not universal, agreement, that term is slowly being replaced in Optics by the word *irradiance*.

The time-averaged value ($T >> \tau$) of the magnitude of the Poynting vector, symbolized by $\langle S \rangle_T$, is a measure of I. In the specific case of harmonic fields and Eq. (3.43),

$$\langle S \rangle_T = c^2\epsilon_0 |\vec{\mathbf{E}}_0 \times \vec{\mathbf{B}}_0| \langle \cos^2 (\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t) \rangle$$

Because $\langle \cos^2(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t) \rangle_T = \frac{1}{2}$ for $T >> \tau$ (see Problem 3.7)

$$\langle S \rangle_T = \frac{c^2\epsilon_0}{2} |\vec{\mathbf{E}}_0 \times \vec{\mathbf{B}}_0|$$

or

$$I \equiv \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2 \qquad (3.44)$$

and

$$I = \epsilon_0 c \langle E^2 \rangle_T \qquad (3.46)$$

Within a linear, homogeneous, isotropic dielectric, the expression for the irradiance becomes

$$I = \epsilon v \langle E^2 \rangle_T \qquad (3.47)$$

Potenza ottica, radianza ed Emittamza…

The time rate of flow of radiant energy is the **optical power** P or **radiant flux**, generally expressed in watts. If we divide the radiant flux incident on or exiting from a surface by the area of the surface, we have the **radiant flux density** (W/m^2). In the former case, we speak of the *irradiance*, in the latter the *exitance*, and in either instance the **flux density**. The irradiance is a measure of the *concentration* of power.

The Inverse Square Law

We saw earlier that the spherical-wave solution of the differential wave equation has an amplitude that varies inversely with r. Let's now examine this same feature within the context of energy conservation.

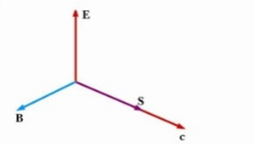


Figura 10.10
Il vettore di Poynting associato ad un'onda elettromagnetica piana.

Mazzoldi - Nigro - Voci
Elementi di Fisica - Elettromagnetismo
EdiSES

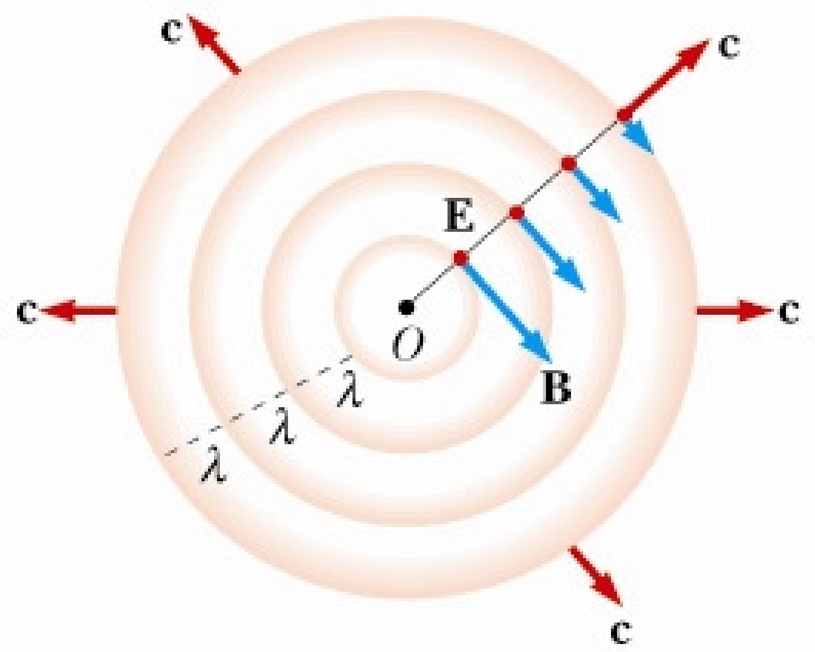


Figura 10.12

Rappresentazione schematica della propagazione del fronte d'onda sferico.

Inverse square law

Let $E_0(r_1)$ and $E_0(r_2)$ represent the amplitudes of the waves over the first and second surfaces, respectively. If energy is to be conserved, the total amount of energy flowing through each surface per second must be equal, since there are no other sources or sinks present. Multiplying I by the surface area and taking the square root, we get

$$r_1 E_0(r_1) = r_2 E_0(r_2)$$

Inasmuch as r_1 and r_2 are arbitrary, it follows that

$$rE_0(r) = \text{constant},$$

and the amplitude must drop off inversely with r. The irradiance from a point source is proportional to $1/r^2$. This is the

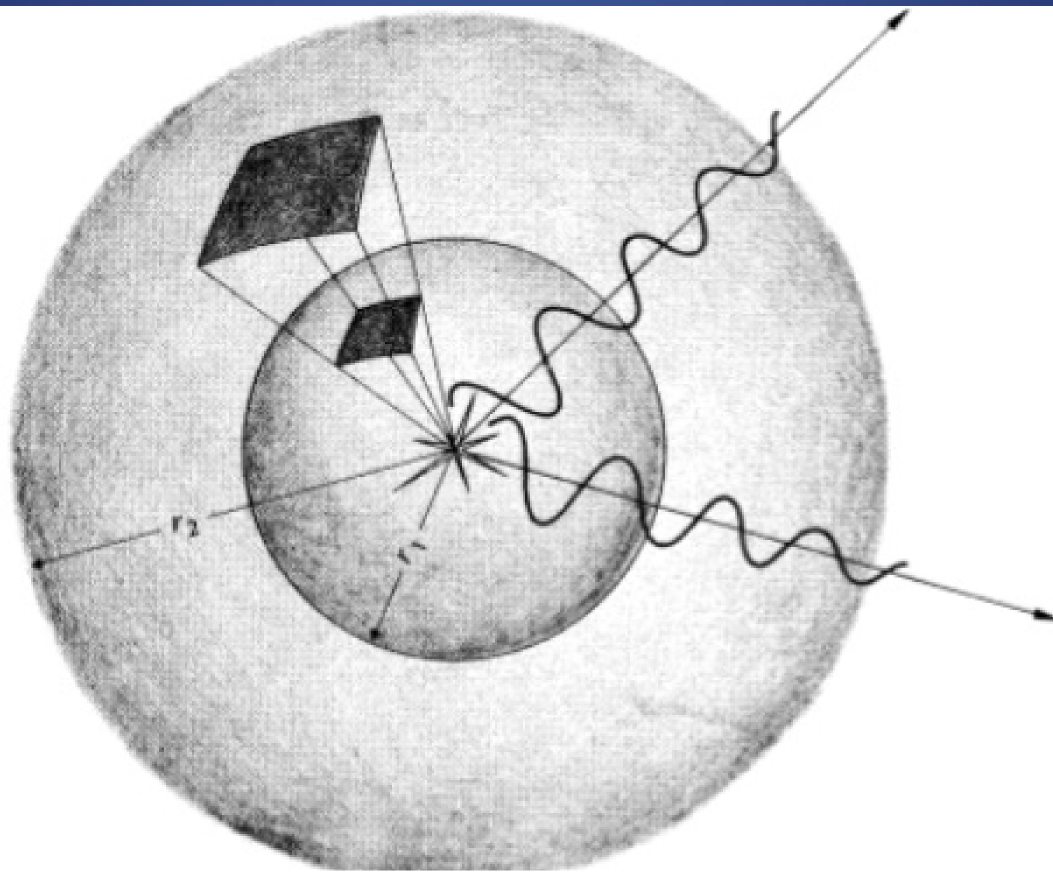
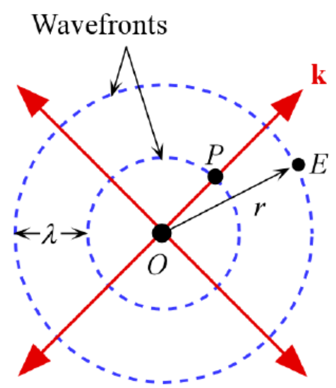


Figure 3.19 The geometry of the inverse square law.

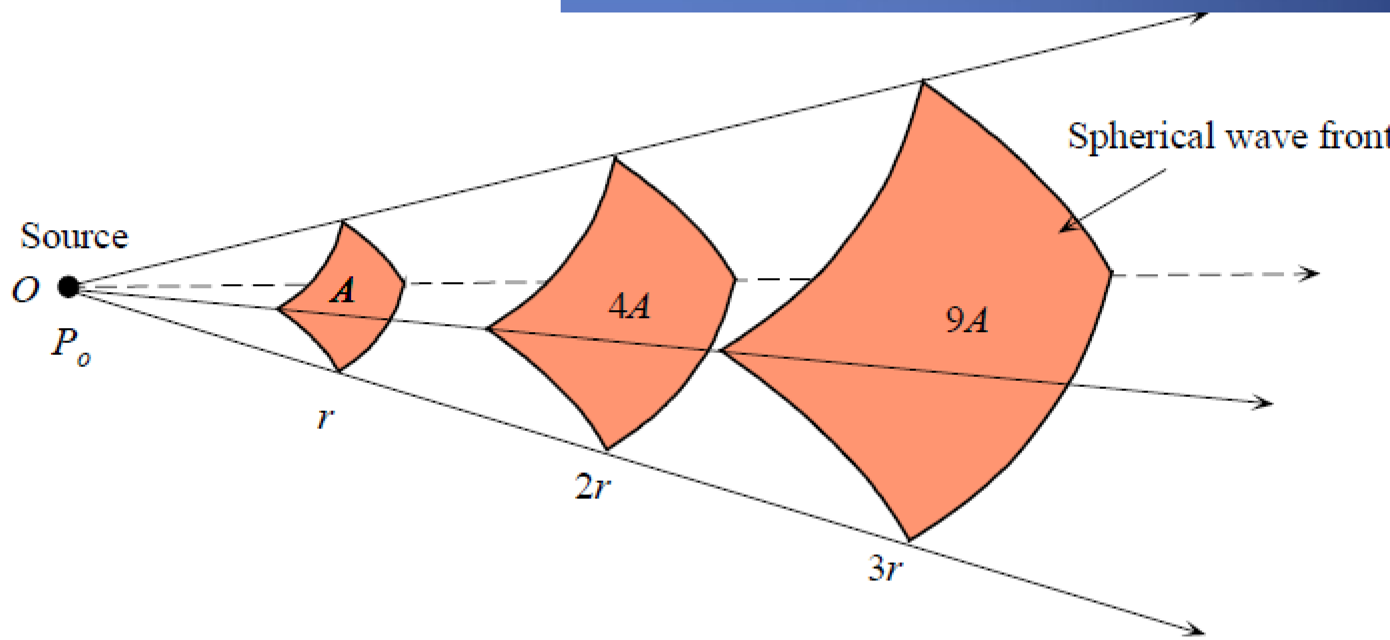
well-known **Inverse Square Law**, which is easily verified with a point source and a photographic exposure meter.

Irradiance of a Spherical Wave

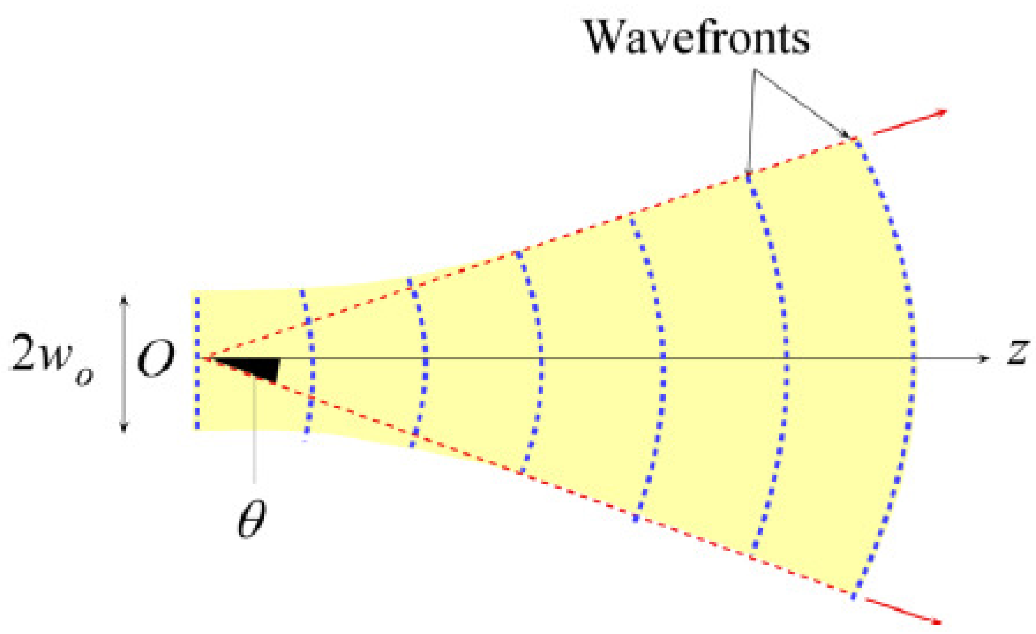


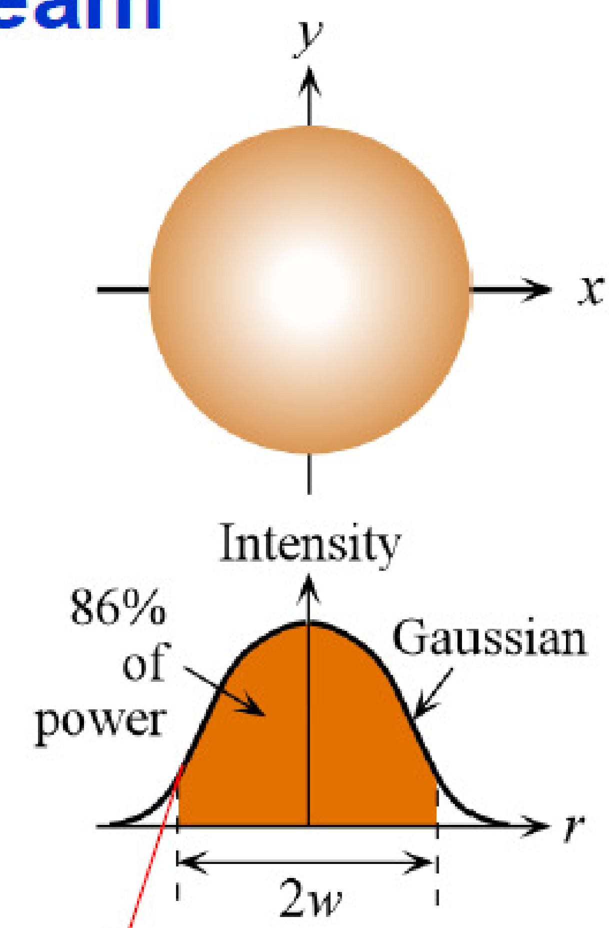
Perfect spherical wave

$$I = \frac{P_o}{4\pi r^2}$$



A Gaussian Beam





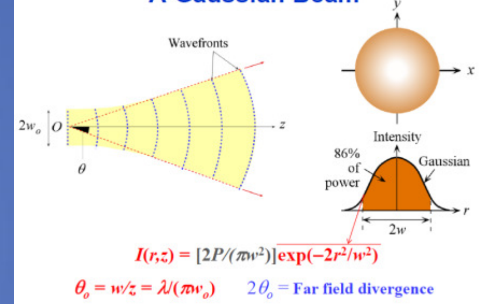
$$I(r,z) = I_o\left(\frac{w_o}{w}\right)^2 \exp(-2r^2/w^2)$$

$\theta_o = w/z = \lambda/(\pi w_o)$ $2\theta_o$ = **Far field divergence**

For a Gaussian beam, the irradiance has a Gaussian distribution across the beam cross-section, as illustrated in Figure 1.5 (b) and (c), and also decreases as the beam propagates along z; the power in the beam becomes spread over larger and larger wavefront surfaces as the wave propagates along z. The irradiance I at a point z from O, and at a radial distance r from the beam axis (Figures 1.5 and 1.6) is given by

$$I(z, r) = I_o\left(\frac{w_o}{w}\right)^2 \exp\left(-\frac{2r^2}{w^2}\right) \quad \textbf{(1.4.8)}$$

A Gaussian Beam



where w_o is the beam waist, w is the beam width at a distance z from O, and I_o is the maximum beam irradiance, which occurs at $z = 0$ when $w = w_o$. Since w depends on z through $2w = 2w_o\left[1 + (z/z_o)^2\right]^{1/2}$, the irradiance also depends on z and I decreases with z. At far away from the Rayleigh range, $z \gg z_o$, the irradiance on the beam axis is

$$I_{\text{axis}}(z) = I_o\frac{z_o^2}{z^2} \quad \textbf{(1.4.9)}$$

which shows that the decay of light intensity with distance is similar to that for a spherical wave. The radial dependence, of course, remains Gaussian.

The total optical power P_o is the EM power carried by a wave, and can be found by integrating the irradiance. For a Gaussian beam P_o and I_o are related by

$$P_o = \frac{1}{2}\left[I_o(\pi w_o^2)\right] \qquad (1.4.10)$$

where it can be seen that the apparent "cross-sectional area" of the beam, πw_o^2, is used to multiply the maximum irradiance with a factor of half to yield the total optical power.

EXAMPLE 1.4.2 Power and irradiance of a Gaussian beam

Consider a 5 mW He-Ne laser that is operating at 633 nm, and has a spot size of 1 mm. Find the maximum irradiance of the beam and the axial (maximum) irradiance at 25 m from the laser.

Solution

The 5 mW rating refers to the total optical power P_o available, and 633 nm is the free-space output wavelength λ. Apply Eq. (1.4.10), $P_o = (1/2)[I_o(\pi w_o^2)]$,

$$5 \times 10^{-3}\,\text{W} = \frac{1}{2} I_o(\pi)\left(\frac{1}{2} \times 1 \times 10^{-3}\,\text{m}\right)^2$$

which gives

$$I_o = 1.273 \times 10^4\,\text{W m}^{-2} = 1.273\,\text{W cm}^{-2}$$

The Rayleigh range z_o was calculated previously as $z_o = \pi w_o^2/\lambda = 1.24$ m in Example 1.1.1. At $z = 25$ m, the axial irradiance is

$$I_{\text{axis}} = (1.273 \times 10^4\,\text{W m}^{-2})\frac{(1.24\,\text{m})^2}{(25\,\text{m})^2} = 31.3\,\text{W m}^{-2} = 3.13\,\text{mW cm}^2$$