# Numerical exercises with PWE2D and GME codes for COST P11 Training School, Nottingham, 18-22 June 2006

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**EXERCISE 1 (2D bands)**

Calculate 2D photonic bands for the triangular lattice of air holes with $\epsilon = 12$, $r/a = 0.45$ and verify that there is a complete photonic band gap for all directions and polarizations between $a/\lambda = 0.4$ and 0.44. Suggested parameters: $N_{\rm pw} = 61$ (rough), $N_{\rm pw} = 109$ (more accurate).

**EXERCISE 2 (2D PhC slab bands)**

Calculate photonic bands in a membrane PhC slab, for the triangular lattice of air holes with $\epsilon = 12$, $d/a = 0.6$, $r/a = 0.45$. Suggested parameters: $N_{\rm pw} = 61$, $N_\alpha = 2$ (rough), $N_{\rm pw} = 109$, $N_\alpha = 4$ (more accurate).
Compare with the results of S.G. Johnson et al., PRB 60, 5751 (1999) (see lecture slides).
Try to answer the following questions:
(a) why is there no complete PBG in the membrane PhC?
(b) why is the value $d/a = 0.6$ considered to be the "optimal" one in PhC slabs with triangular lattice, as stated in the paper by Johnson et al?
See also IEEE-JQE 38, 891 (2002).

**EXERCISE 3 (comparison with effective waveguide)**

Calculate photonic bands for horizontally even (TE-like) modes in a membrane PhC slab, for the triangular lattice of air holes, with the parameters $\epsilon = 12$, $d/a = 0.5$, $r/a = 0.3$.
Compare the photonic band dispersion with that of the effective waveguide.

**EXERCISE 4 (comparison with SOI, effect of slab asymmetry)**

Calculate photonic bands in a SOI PhC slab whose layers have dielectric constants $\epsilon_1 = 1$, $\epsilon_2 = 12$, $\epsilon_3 = 2.1$ for the triangular lattice of air holes with $d/a = 0.5$, $r/a = 0.3$. Compare the bands with those of Exercise 3. What are the consequences of having an asymmetric PhC slab?

Additionally: compare the bands in the SOI PhC slab with those calculated in a "symmetrized" waveguide with $\epsilon_1 = \epsilon_3 = 1.5$, $\epsilon_2 = 12$. This approximation is useful for identifying modes which are dominantly TE-like or TM-like, in an asymmetric structure.

**EXERCISE 5 (losses in 1D PhC slabs)**

Calculate photonic bands and $\mathrm{Im}(\omega)$ of vertically odd (TE) modes in a membrane PhC slab patterned with a 1D lattice: $\epsilon = 12$, $d/a = 0.2$, $r/a = 0.3$. Use jparxy=-1 (both horizontal parities) and jparkz=+1 (vertically odd modes, which are purely TE for the case of a 1D lattice). Plot the losses as a function of frequency, look at the behaviour of losses as a function of wavevector and band index.

Additionally: Calculate photonic bands and $\mathrm{Im}(\omega)$ for a SOI PhC slab ($\epsilon_1 = 1$, $\epsilon_2 = 12$, $\epsilon_3 = 2.1$) with the same 1D lattice. Notice that the losses for each band are slightly higher for the asymmetric structure. What is the effect of slab asymmetry? See also PRE 69, 056603 (2004).

**EXERCISE 6 (losses in 2D PhC slabs)**

Calculate photonic bands and $\mathrm{Im}(\omega)$ of horizontally even (TE-like) modes in a membrane PhC slab patterned with a triangular lattice of air holes: $\epsilon = 12$, $d/a = 0.5$, $r/a = 0.3$, jparxy=0. Consider only the $\Gamma$K symmetry direction and separate photonic modes according to vertical parity. Plot the losses as a function of frequency, compare with the results of PRB 73, 235114 (2006) and also with T. Ochiai and K. Sakoda, PRB 63, 125107 (2001) (see lecture slides).

N.b. It is possible to plot $\mathrm{Im}(\omega)$ as a function of wavevector, as done in the above papers... but then it is more cumbersome to associate the losses with each corresponding band.

**EXERCISE 7 (defect modes in W1 waveguides)**

Calculate the dispersion of the line-defect modes in a W1 waveguide (missing row of holes in the ΓK direction of the triangular lattice) realized in a high-index membrane with $\epsilon = 12$, $d/a = 0.5$, $r/a = 0.3$. Calculate only horizontally even modes (jparxy=0) and separate the modes according to vertical parity (jparkz=0 or 1).

A supercell in the ΓM direction has to be introduced: setting $w0 = \sqrt{3}a$, the channel width is alength2*w0 while the period in the Γ-M direction is (alength2+alength3)*w0.

Suggested parameters are alength3=4, $N_{\rm pw} = 91$, $N_\alpha = 1$ (very rough), $N_{\rm pw} = 161$, $N_\alpha = 2$ (rough). When changing the supercell period, the number of plane waves has also to be adjusted $\rightarrow$ keep a constant cutoff.

Notice the the index-guided is odd under vertical parity, while the gap-guided mode is even. This is because the modes are quasi-TE, i.e., the dominant electric field component is perpendicular to the line defect: thus the jparkz=+1 or index-guided mode has a spatially even electric field, but is globally odd under vertical mirror symmetry. This is the most important defect mode for applications, because it has a dispersion region below the light line with high group velocity and low losses. See e.g. A. Chutinan and S. Noda, PRB 62, 4488 (2000); S.G. Johnson et al., PRB 62, 8212 (2000) and lecture slides.

Additionally: try plotting the field components of the defect modes.

**EXERCISE 8 (propagation loss of line-defect mode in a W1 waveguide)**

Calculate propagation losses of the line-defect mode in a W1 waweguide realized in a high-index membrane with $\epsilon = 12$, $d/a = 0.5$, $r/a = 0.3$. Consider only horizontally even (jparxy=0) and vertically odd (jparkz=1) modes, according to the results of Exercise 7. A supercell in the ΓM direction has to be introduced: setting $w0 = \sqrt{3}a$, the channel width is alength2*w0 while the period in the ΓM direction is (alength2+alength3)*w0.

Plot Im($\omega$), the group velocity $v_g = d\omega/dk$ and the propagation loss $4.34 * 2 * \mathrm{Im}(k)$ as a function of frequency. The code yields the loss in decibel/lattice constant: in order to get the propagation loss in dB/mm, use a typical lattice constant $a = 420$ nm (yielding the defect-mode wavelength around 1.55 micron).

Suggested parameters are alength3=4, $N_{\mathrm{pw}} = 91$, $N_\alpha = 1$ (very rough), $N_{\mathrm{pw}} = 161$, $N_\alpha = 2$ (rough). When changing the supercell period, the number of plane waves has also to be adjusted → keep a constant cutoff.

The jumps in Im($\omega$) and in the propagation loss are unphysical, they arise because of the finite supercell width. In order to get the limit of an isolated defect with (relatively) smooth curves, an average over different supercell periods has to be performed, as in APL 82, 2011 (2003).

Notice that a typical value of propagation loss for the line-defect mode above the light line is around 50 dB/mm: this is much too high for applications. The useful low-loss region is below the light line, where diffraction losses are purely extrinsic as they depend on the presence of fabrication disorder.