

# ONE THING THAT GENERAL RELATIVITY SAYS ABOUT PHOTONS IN MATTER

S. ANTOCI AND L. MIHICH

ABSTRACT. Let us abandon for a moment the strict epistemological standpoint of quantum field theory, that eventually comes to declare nonsensical any question about the photon posed outside the quantum theoretical framework. We can then avail of the works by Whittaker et al. and by Synge about the particle and the wave model of the photon in the vacuum of general relativity. We can also rely on important results found by Gordon and by Pham Mau Quan: thanks to Gordon's discovery of an effective metric these authors have been able to reduce to the vacuum case several problems of the electromagnetic theory of dielectrics.

The joint use of these old findings allows one to conclude that a quantum theoretical photon in an isotropic dielectric has a classical simile only if the dielectric is also homogeneous.

## 1. INTRODUCTION

When writing a paper with the title given above, one is fully aware of the fact that it will be judged very differently by readers with different epistemological inclinations. Imagine that we strictly embrace what Schrödinger, in a memorable paper [1], dubbed “Der bewußte Wechsel des erkenntnistheoretischen Standpunkts”, and thereby “wir haben unsere naiv-realistische Unschuld verloren”. Photons are quantum theoretical entities, and the positivistic credo currently attached to quantum theory allows us to ask only questions that deal with quantities that can be observed, at least in principle, according to the quantum framework that we have adopted. Hence there is no room for the good old “Physik der Modelle” of the present writing, despite the fundamental role that such a kind of physics has played in fostering the very birth of quantum mechanics and of quantum field theory. For us, this sort of physics shall be simply incompetent to say anything on the subject, for it lies outside the allowed conceptual framework.

If, on the contrary, we still have not yet completely lost our ingenuous-realistic innocence, we may feel relieved if the problem that we should tackle quantum mechanically has a classical simile. In that case, we can hope to rely on some clear-cut argument of classical origin for guiding our steps if

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*Key words and phrases.* General relativity, electrodynamics, geometrical optics, light quanta.

the quantum mechanical apparatus, despite its programmatic self-reliance, for some reason turns out to be inept at producing an answer whatsoever.

This lamentable occurrence happens when one attempts to deal with quantum optics in dielectric media. As recently stressed [2], the usual presentations rely on *ad hoc* effective Hamiltonians drawn in some way or other from the classical electromagnetism of continua, thereby avoiding the admittedly very hard, but necessary task of *deriving* an effective Hamiltonian from a fully quantum description of the whole system consisting of matter plus field. No wonder then, if after more than five decades from the first attempt at producing “phenomenological” photons in homogeneous, isotropic matter [3], [4], a unique, satisfactory answer is still lacking. One is still offered with diverging options. If one starts from Minkowski’s energy tensor [5], like Jauch and Watson did, one is presented with photons endowed with an unpalatable *spacelike* momentum-energy four-vector. If one starts instead from Abraham’s energy tensor [6], as both strong theoretical arguments [7] and sound experimental evidence [8, 9] might suggest to do, one reaches the conclusion that photons in dielectric matter can not be defined, because the energy operator and the momentum operator have no common eigenvectors; furthermore, both the field energy and the field momentum are not constant in time [4]. This seemingly odd result is instead quite reasonable, for Abraham’s tensor density has a nonvanishing four-divergence also in the absence of charges and currents, thereby entailing a continuous exchange of energy and momentum with the dielectric medium. Only by extracting from Abraham’s tensor a so called radiation tensor [10] with vanishing four-divergence did Nagy succeed [4] in producing photons in dielectric matter endowed with the expected timelike momentum-energy four-vector.

Since the subsequent works, quoted e.g. in ref. [2], have not led to a solution of the problem at issue, the reader with an ingenuous-realistic epistemological inclination may find interesting to learn what “die Physik der Modelle” has to say about photons in matter. For this reader we shall recall the existence of two very important results obtained many years ago. One was found by Whittaker et al. [11] and by Synge [12], the second one is already contained in the seminal paper by Gordon [7] and was later retrieved and extended [13] by Pham Mau Quan in a much simpler way through the theory of characteristics; both are now practically forgotten<sup>1</sup>. The joint use of these achievements of the past allows one to draw conclusions about the very existence of photons in isotropic dielectrics.

## 2. SYNGE’S PHOTONS IN THE VACUUM OF GENERAL RELATIVITY

“Die Physik der Modelle” has no room for such esoteric ideas as the wave-particle dualism, since things are supposed to exist in spacetime in a way totally independent of any act of measurement. Nothing however

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<sup>1</sup>There has been a recent upsurge of interest in Gordon’s effective metric. See e.g. [14] and references contained therein.

forbids to think that one and the same physical reality may be accounted for by more than one theoretical model. Therefore it is totally legitimate to check whether a given classical theory, let us say general relativity, can contain both a wave description and a particle description of the photon, and to investigate how the characteristic quantities of both descriptions are mutually related. This is just what the authors of refs. [11] and [12] have done for the photons in the vacuum of general relativity. Due to its masterful clearness, we shall report here Synge's account *in extenso*.

If the atoms are treated as points, it is necessary that the photon be pictured as something capable of being emitted and absorbed by points. There is a venerable line of thought that leads to assume that the world-line of a photon is a null geodesic. In fact the forerunners of the photons, i.e. the light rays of geometrical optics, were postulated to be null geodesics of the spacetime metric  $g_{ik}$  already in 1917 by Hilbert [15]. The postulate was then shown to be a mere consequence of the electromagnetic equations in a gravitational field by von Laue [16]; the same result was later reproduced by Whittaker in a two-page note [17] that relies on the theory of characteristics. Synge was thus led to assume that the world line of a photon satisfies the equations

$$(2.1) \quad \frac{\delta}{\delta u} \frac{dx^i}{du} = 0, \quad g_{ik} \frac{dx^i}{du} \frac{dx^k}{du} = 0,$$

where  $\delta/\delta u$  stands for the absolute derivative with respect to the special parameter  $u$ .

If the photon can be modeled as a point particle, we shall attribute to it a momentum-energy vector  $M^i$  that is tangent to the photon's world line and is parallelly propagated along it. One therefore poses:

$$(2.2) \quad M^i = \theta \frac{dx^i}{du}, \quad \frac{\delta M^i}{\delta u} = 0,$$

where  $\theta$ , assumed to be positive, happens to be a constant thanks to (2.1). Since a photon is supposed to travel with the speed of light, it must have zero proper mass. Therefore it must be

$$(2.3) \quad M_i M^i = 0;$$

the index of  $M^i$  has been lowered with the metric  $g_{ik}$ ; we choose by convention that the Minkowski form to which the latter can be transformed locally is  $\eta_{ik} \equiv \text{diag}(1, 1, 1, -1)$ . We shall model an atom like a timelike point particle endowed with the momentum-energy vector

$$(2.4) \quad m_0 u^i = m_0 \frac{dx^i}{ds}, \quad ds^2 \equiv -g_{ik} dx^i dx^k;$$

$m_0$  is the atom's rest mass or rest energy, while  $u^i$  is its four-velocity. The energy collected by an atom through the absorption of a photon can be

calculated by postulating that the absorption process is ruled by the conservation law:

$$(2.5) \quad m'_0 u'^i = m_0 u^i + M^i,$$

where  $m'_0$  is the rest mass, and  $u'^i$  is the four-velocity of the atom after the absorption of the photon. By rewriting the conservation law in covariant form and by multiplying the two forms term by term one finds

$$(2.6) \quad -m'^2_0 = -m^2_0 + 2m_0 M_i u^i,$$

i.e.

$$(2.7) \quad \frac{m'^2_0 - m^2_0}{2m_0} = -M_i u^i.$$

For optical processes  $(m'_0 - m_0)/m_0$  is very small with respect to unity, and this fact suggests defining the energy  $E$  of a photon with momentum-energy vector  $M^i$  relative to a pointlike “atom” endowed with four-velocity  $u^i$  as

$$(2.8) \quad E = -M_i u^i.$$

But another, equally time-honoured tradition leads to think of a photon as a wavelike phenomenon and to draw, with Synge, the spacetime diagram of Figure 1. In the wave model the photon is not pictured by just one null world-line, but by the two-dimensional ribbon  $A_0 B_0 B A$ , delimited by the null geodesics  $A_0 A$ ,  $B_0 B$ , and by the line elements  $ds_0 = A_0 B_0$ ,  $ds = AB$ , taken on the world-lines of the emitting and of the absorbing atom respectively.

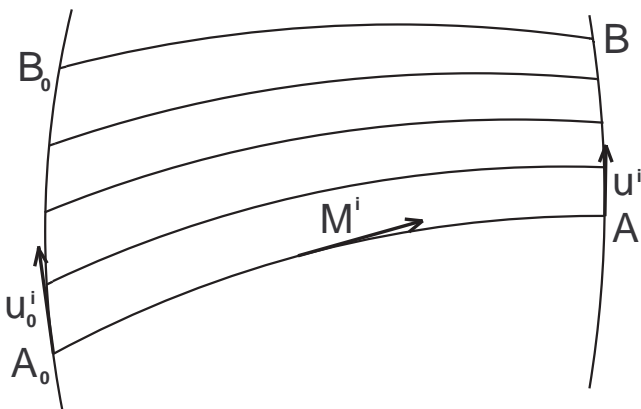


FIGURE 1. Spacetime diagram of Synge’s “photon”.

To each null world-line within the ribbon it corresponds a phase of the wave process. The null lines drawn in the figure have, say, zero phase. If these lines are  $n+1$  in number, it means that the emission process entails  $n$  periods, and  $n$  too shall be the number of the periods occurring in the absorption process.

Therefore the frequency of emission, defined as the number of periods per unit proper time along the world-line of the emitting atom, shall read (with an ugly but otherwise innocuous licence in the notation)

$$(2.9) \quad \nu_0 = n/ds_0,$$

while the frequency of absorption, defined with respect to the world line of the absorbing atom, turns out to be

$$(2.10) \quad \nu = n/ds.$$

By comparing these two equations one finds

$$(2.11) \quad \nu_0 ds_0 = \nu ds,$$

i.e. the relation governing the spectral shift in a gravitational field, whose quite general, geometrical derivation was first given in 1923 by Lanczos [18].

The special parameter  $u$  must be chosen on each of the null geodesics of the ribbon  $A_0B_0BA$ . Since each of the  $u$ 's is given up to a linear transformation  $u' = au + b$ , where  $a$  and  $b$  are constants that can be arbitrarily fixed on each null geodesic, we can freely assume that these parameters have a single starting value on  $A_0B_0$  and a single terminal value on  $AB$ . If  $v$  is another parameter, taken to be constant along the single null geodesic, the 2-space of the ribbon can be described by the equation

$$(2.12) \quad x^i = x^i(u, v),$$

while of course the partial differential equation

$$(2.13) \quad g_{ik} \frac{\partial x^i}{\partial u} \frac{\partial x^k}{\partial u} = 0$$

needs to be satisfied. We are now equipped for considering further the particle model of the photon and for studying the relation between the energy release at the atom on the left side of the figure and the energy intake occurring at the atom on the right side. If  $M^i$  is the momentum-energy vector of the photon, we can write

$$(2.14) \quad \frac{\partial}{\partial u} \left( M_i \frac{\partial x^i}{\partial v} \right) = \frac{\delta M_i}{\delta u} \frac{\partial x^i}{\partial v} + M_i \frac{\delta}{\delta u} \frac{\partial x^i}{\partial v}.$$

We have also

$$(2.15) \quad \frac{\delta}{\delta u} \frac{\partial x^i}{\partial v} \equiv \frac{\partial^2 x^i}{\partial u \partial v} + \Gamma_{kl}^i \frac{\partial x^k}{\partial v} \frac{\partial x^l}{\partial u} = \frac{\delta}{\delta v} \frac{\partial x^i}{\partial u},$$

where  $\Gamma_{kl}^i$  is the Christoffel connection built with  $g_{ik}$ . Since  $\delta M_i/\delta u = 0$ , the right hand side of (2.14) can be rewritten as

$$(2.16) \quad M_i \frac{\delta}{\delta u} \frac{\partial x^i}{\partial v} = \theta g_{ik} \frac{\partial x^k}{\partial u} \frac{\delta}{\delta v} \frac{\partial x^i}{\partial u} = \frac{1}{2} \theta \frac{\partial}{\partial v} \left( g_{ik} \frac{\partial x^i}{\partial u} \frac{\partial x^k}{\partial u} \right) = 0.$$

Hence one finds

$$(2.17) \quad \frac{\partial}{\partial u} \left( M_i \frac{\partial x^i}{\partial v} \right) = 0,$$

and, with reference to Figure 1:

$$(2.18) \quad \left( M_i \frac{\partial x^i}{\partial v} \right)_{A_0} = \left( M_i \frac{\partial x^i}{\partial v} \right)_A .$$

Let  $u_0^i$ ,  $u^i$  be the four-velocities of the atoms at  $A_0$  and at  $A$  respectively. If  $dv$  is the infinitesimal increment of the parameter  $v$  when going from the null geodesic  $A_0A$  to the neighbouring one  $B_0B$ , we can write:

$$(2.19) \quad \left( \frac{\partial x^i}{\partial v} \right)_{A_0} dv = u_0^i ds_0, \quad \left( \frac{\partial x^i}{\partial v} \right)_A dv = u^i ds,$$

hence from (2.18) we get

$$(2.20) \quad (M_i)_{A_0} u_0^i ds_0 = (M_i)_A u^i ds.$$

But the definition (2.8) of the energy of a photon absorbed (or emitted) by an atom allows to rewrite the last equation as just

$$(2.21) \quad E_0 ds_0 = E ds.$$

This is a really momentous result: if we divide term by term this equation, derived by considering the photon as a particle, and equation (2.11), that was obtained by considering the photon as a wave, we get eventually that it must be:

$$(2.22) \quad \frac{E_0}{\nu_0} = \frac{E}{\nu}.$$

Therefore, general relativity contains both a wave model and a particle model of the photon *in vacuo*, and the relation between the two models is such that when a photon is emitted by one atom and absorbed by another one in presence of a gravitational field, the ratio energy/frequency is the same for emission and for absorption. This ratio is independent of the behaviour of the gravitational field and of the state of motion of the two atoms.

### 3. GORDON'S REDUCTIO AD VACUUM OF THE ELECTROMAGNETIC THEORY FOR A HOMOGENEOUS, ISOTROPIC DIELECTRIC

In 1923 Walter Gordon showed [7] that Maxwell's equations for matter that is homogeneous and isotropic when considered in its rest frame can be rewritten as Maxwell's equations *in vacuo*, provided that we adopt, in writing those equations, not the true spacetime metric  $g_{ik}$ , but the effective metric

$$(3.1) \quad \sigma_{ik} = g_{ik} + \left( 1 - \frac{1}{\epsilon\mu} \right) u_i u_k$$

in which  $\epsilon$ ,  $\mu$  are respectively the dielectric constant and the magnetic permeability of the dielectric medium, while  $u^i$  is its four-velocity. This is a remarkable finding with far reaching consequences, and we shall spend some words to show how it comes about. By following the established convention [19], we represent the electric displacement and the magnetic field by the

antisymmetric, contravariant tensor density  $\mathbf{H}^{ik}$ , while the electric field and the magnetic induction are accounted for by the skew, covariant tensor  $F_{ik}$ . With these geometrical objects we define the four-vectors:

$$(3.2) \quad F_i = F_{ik}u^k, \quad H_i = H_{ik}u^k,$$

where  $u^i$  is the four-velocity of matter. In general relativity a linear electromagnetic medium can be told to be homogeneous and isotropic in its rest frame if its constitutive equation reads

$$(3.3) \quad \mu H^{ik} = F^{ik} + (\epsilon\mu - 1)(u^i F^k - u^k F^i).$$

Gordon noticed that (3.3) can be rewritten as

$$(3.4) \quad \mu H^{ik} = [g^{ir} - (\epsilon\mu - 1)u^i u^r] [g^{ks} - (\epsilon\mu - 1)u^k u^s] F_{rs},$$

and, since the contravariant form of the effective metric tensor (3.1) is

$$(3.5) \quad \sigma^{ik} = g^{ik} - (\epsilon\mu - 1)u^i u^k,$$

the constitutive equation can be rewritten as

$$(3.6) \quad \mu \mathbf{H}^{ik} = \sqrt{g} \sigma^{ir} \sigma^{ks} F_{rs},$$

where  $g \equiv -\det(g_{ik})$ . With some simple algebra one finds [7] that

$$(3.7) \quad \sigma = \frac{g}{\epsilon\mu},$$

where  $\sigma \equiv -\det(\sigma_{ik})$ . Hence (3.3) can be eventually rewritten as

$$(3.8) \quad \mathbf{H}^{ik} = \sqrt{\frac{\epsilon}{\mu}} \sqrt{\sigma} \sigma^{ir} \sigma^{ks} F_{rs}.$$

Therefore, apart from the constant factor  $\sqrt{\epsilon/\mu}$ , the constitutive equation for this dielectric medium is just the same as the one occurring for a general relativistic vacuum. All the equations and all the theorems that hold for electromagnetism in the vacuum case will apply also to our medium, provided that  $\sigma_{ik}$  be substituted for  $g_{ik}$  in the original vacuum equations. In particular, we see that the results [16, 17] found by von Laue and by Whittaker for the propagation of light rays in the vacuum of general relativity immediately apply to our dielectric medium: in the limit of geometrical optics, the light rays in a dielectric that is homogeneous and isotropic in its local rest frame shall be [7] the null geodesics of an ‘‘effective spacetime’’ endowed with the metric (3.1).

Gordon’s idea of the effective metric was later resumed by Pham Mau Quan [13]. By availing of the theory of characteristics, he could extend Gordon’s just quoted result to dielectrics that are isotropic, but not homogeneous, since  $\epsilon$  and  $\mu$  are assigned functions of the coordinates  $x^i$ .

#### 4. THE RELATION BETWEEN THE WAVE AND THE PARTICLE MODEL OF THE PHOTON IN AN ISOTROPIC DIELECTRIC

Thanks to the findings of Whittaker, Synge, Gordon and Pham Mau Quan recalled in the two previous sections, we can build both a wave and a particle model of the photon in an isotropic dielectric and investigate how the two models are mutually related in the new situation.

The wave model of the photon in an isotropic dielectric, i.e. in a medium with the constitutive equation (3.3), where  $\epsilon$  and  $\mu$  have an assigned dependence on  $x^i$ , needs no explanation. The spacetime diagram of Figure 1 can be drawn as it stands also in the case of the isotropic dielectric, and we find that the relation between the frequencies of the photon measured on the world lines of the emitting and of the absorbing atoms is still given by equation (2.11). Let us only emphasize that the line elements  $ds_0$  and  $ds$  measure the proper time of the pointlike atoms in the actual spacetime and are therefore defined by the true spacetime metric  $g_{ik}$ .

The particle model of the photon entails instead important changes, that are however both mandatory and natural, if one keeps in mind the equivalence between the propagation in the dielectric and the propagation in a general relativistic vacuum with the effective metric  $\sigma_{ik}$  given by equation (3.1). We know from [13] that the light rays in the dielectric are null geodesics with respect to that metric. The definition of these geodesics is no longer provided by equation (2.1). It reads instead:

$$(4.1) \quad \frac{\sigma \delta dx^i}{\delta u du} \equiv \frac{d^2 x^i}{du^2} + \Sigma_{kl}^i \frac{dx^k}{du} \frac{dx^l}{du} = 0$$

*cum*

$$(4.2) \quad \sigma_{ik} \frac{dx^i}{du} \frac{dx^k}{du} = 0;$$

we shall henceforth denote with  $\sigma \delta / \delta u$  the absolute derivative performed with the affine connection

$$(4.3) \quad \Sigma_{kl}^i = \frac{1}{2} \sigma^{im} (\sigma_{mk,l} + \sigma_{ml,k} - \sigma_{kl,m});$$

$u$  is again a special parameter. If the photon in the dielectric can be modeled as a point particle, we shall attribute to it a momentum-energy vector  $M^i$  tangent to its world line, defined by (4.1) and (4.2); the equivalence with the vacuum case forces us to assume that  $M^i$  is parallelly propagated along that world line. One therefore still poses

$$(4.4) \quad M^i = \theta \frac{dx^i}{du},$$

but now, instead of  $\delta M^i / \delta u = 0$ , one shall write

$$(4.5) \quad \frac{\sigma \delta M^i}{\delta u} = 0.$$



Thanks to (4.1) the scalar  $\theta$ , assumed to be positive, happens again to be a constant. This photon travels with the speed of light in the spacetime with the metric  $\sigma_{ik}$ ; hence in this “effective spacetime” it must exhibit zero proper mass. Therefore it must be:

$$(4.6) \quad M_{(i)}M^i \equiv \sigma_{ik}M^iM^k = 0;$$

we adopt henceforth Gordon’s convention of enclosing within round parentheses the indices moved with the effective metric  $\sigma_{ik}$ . Let us now look at this photon in the true spacetime, the one with  $g_{ik}$  as metric tensor. Equation (4.6) can be rewritten as

$$(4.7) \quad \begin{aligned} M_{(i)}M^i &= \left[ g_{ik} + \left( 1 - \frac{1}{\epsilon\mu} \right) u_i u_k \right] M^i M^k \\ &= M_i M^i + \left( 1 - \frac{1}{\epsilon\mu} \right) (u_i M^i)^2 = 0, \end{aligned}$$

When measured with the metric  $g_{ik}$  of the true spacetime the momentum-energy vector  $M^i$  will not be a null one, and with some trepidation we now check whether it is spacelike, as it occurs with the photon obtained by Jauch and Watson in their phenomenological quantum electrodynamics [3], or whether it is timelike, as Nagy instead found [4], and as it is required by the consistency of the particle model. Equation (4.7) says that

$$(4.8) \quad M_i M^i = \left( \frac{1}{\epsilon\mu} - 1 \right) (u_i M^i)^2,$$

and since  $\epsilon\mu > 1$ , one finds that  $M_i M^i < 0$ . Therefore, with our sign convention for the metric, the momentum-energy vector of the photon happens to be timelike; the particle model is a consistent one, and the photon in the dielectric behaves like an ordinary massive point particle, for which a rest frame exists.

As a consequence we can repeat here, with the due changes, the argument leading to equation (2.21). We start again from the conservation equation (2.5) that rules the absorption process of a photon by an atom; we shall assume that it holds, in unaltered form, also in the dielectric. However, if we lower the indices of (2.5) with  $g_{ik}$  and multiply term by term the contravariant and the covariant form of the conservation law we do not get (2.6), since  $M_i M^i$  no longer vanishes. One gets instead

$$(4.9) \quad -m_0'^2 = -m_0^2 + 2m_0 M_i u^i + \left( \frac{1}{\epsilon\mu} - 1 \right) (u_i M^i)^2;$$

however for optical processes the last term is negligible with respect to the other ones. The form (2.8) can still be used to define, with the same degree of approximation as in the vacuum case, the energy  $E$  of a photon with momentum-energy vector  $M^i$  relative to a pointlike “atom” endowed with four-velocity  $u^i$ . We can avail of Figure 1 also for dealing with the photon as

a particle, and we retain the parametrisation of the “ribbon”  $A_0B_0BA$  exactly as it stands in the vacuum case. Since now equation (4.5) is substituted for the second equation (2.2), let us consider the quantity

$$(4.10) \quad \frac{\partial}{\partial u} \left( M_{(i)} \frac{\partial x^i}{\partial v} \right) = \frac{\sigma \delta M_{(i)}}{\delta u} \frac{\partial x^i}{\partial v} + M_{(i)} \frac{\sigma \delta}{\delta u} \frac{\partial x^i}{\partial v};$$

the first term at the right hand side vanishes due to (4.5). We have also

$$(4.11) \quad \frac{\sigma \delta}{\delta u} \frac{\partial x^i}{\partial v} \equiv \frac{\partial^2 x^i}{\partial u \partial v} + \Sigma_{kl}^i \frac{\partial x^k}{\partial v} \frac{\partial x^l}{\partial u} = \frac{\sigma \delta}{\delta v} \frac{\partial x^i}{\partial u},$$

hence we obtain

$$(4.12) \quad M_{(i)} \frac{\sigma \delta}{\delta u} \frac{\partial x^i}{\partial v} = \theta \sigma_{ik} \frac{\partial x^k}{\partial u} \frac{\sigma \delta}{\delta v} \frac{\partial x^i}{\partial u} = \frac{1}{2} \theta \frac{\partial}{\partial v} \left( \sigma_{ik} \frac{\partial x^i}{\partial u} \frac{\partial x^k}{\partial u} \right) = 0$$

due to (4.2). Therefore

$$(4.13) \quad \frac{\partial}{\partial u} \left( M_{(i)} \frac{\partial x^i}{\partial v} \right) = 0$$

and, again with reference to Figure 1:

$$(4.14) \quad \left( M_{(i)} \frac{\partial x^i}{\partial v} \right)_{A_0} = \left( M_{(i)} \frac{\partial x^i}{\partial v} \right)_A.$$

The same argument as in Section 2 leads to write now

$$(4.15) \quad (M_{(i)} u^i)_{A_0} ds_0 = (M_{(i)} u^i)_A ds.$$

Since

$$(4.16) \quad \begin{aligned} M_{(i)} u^i &\equiv \sigma_{ik} M^i u^k = \left[ g_{ik} + \left( 1 - \frac{1}{\epsilon \mu} \right) u_i u_k \right] M^i u^k \\ &= M_i u^i - \left( 1 - \frac{1}{\epsilon \mu} \right) M_i u^i = \frac{M_i u^i}{\epsilon \mu}, \end{aligned}$$

instead of (2.20) one finds

$$(4.17) \quad \left( \frac{M_i u^i}{\epsilon \mu} \right)_{A_0} ds_0 = \left( \frac{M_i u^i}{\epsilon \mu} \right)_A ds,$$

hence one eventually obtains that the ratio, say, between the energy emitted by the atom on the left side of Figure 1 and the energy absorbed by the atom on the right side is no longer given by equation (2.21), since now

$$(4.18) \quad \left( \frac{E}{\epsilon \mu} \right)_{A_0} ds_0 = \left( \frac{E}{\epsilon \mu} \right)_A ds,$$

i.e. the ratio depends on the values that  $\epsilon$  and  $\mu$  happen to assume at the spacetime location of the emitting and of the absorbing atom respectively. Since, however, the relation (2.11) between the frequencies does not undergo

any change when going from the vacuum case to the case of the isotropic dielectric, instead of the vacuum equation (2.22) we get

$$(4.19) \quad \left( \frac{E}{\nu\epsilon\mu} \right)_{A_0} = \left( \frac{E}{\nu\epsilon\mu} \right)_A.$$

## 5. CONCLUSION

The last equation summarizes one thing that general relativity has to say about photons in isotropic dielectric matter, provided that its “Physik der Modelle” is not declared *a priori* incompetent to deal with such a definitely quantum affair. In the case of the general relativistic vacuum, investigated by Whittaker et al. [11] and by Synge [12], the particle and the wave model of the photon are mutually related in a way that, at least for optical photons, has an important point of agreement with the quantum theory of radiation. One should not be as naive as to identify concepts stemming from so different theoretical constructions like general relativity and quantum field theory just because these concepts have been christened with the same name in both theories. But also with this proviso it is indeed remarkable that in general relativity the ratio of the two invariant quantities  $E$  and  $\nu$ , associated to the photon as shown in Section 2, has at the absorption just the same value that it exhibits at the emission. This ratio is totally independent of the way the photon travels from the emitting to the absorbing atom, and is also independent of the state of motion of these atoms. A theoretician that still has a penchant for an ingenuous-realistic epistemology can feel enticed by these findings into taking very seriously the concept of photon *in vacuo*, in keeping with Einstein’s famous complaint<sup>2</sup>.

The same theoretician will draw from the result of Section 4, expressed in equation (4.19), the following hints. If the dielectric is homogeneous and isotropic, it is likely that the program [2] of deriving an effective Hamiltonian from a fully quantum description of the whole system consisting of matter plus field will be worth the inordinate effort it presumably requires. In fact the classical wave and the classical particle models do exist and show that in such a medium the ratio of the invariants  $E$  and  $\nu$  will be the same at the emission and at the absorption, just as it happens *in vacuo*. For a homogeneous, isotropic dielectric the very existence of “phenomenological photons” derived by quantization from the effective Hamiltonian mentioned above is therefore an expected occurrence, since a classical simile is already known to exist.

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<sup>2</sup> Letter to Michele Besso of December 12, 1951: “The whole fifty years of conscious brooding have not brought me nearer to the answer to the question ‘What are light quanta?’. Nowadays every scalawag believes that he knows what they are, but he deceives himself” [20]. English translation by J. Stachel [21].

If the dielectric is not homogeneous, however, the same equation (4.19) shows that the constancy of the ratio between  $E$  and  $\nu$  is no longer ensured. Therefore, for such a medium, a classical simile of the quantum theoretical photon cannot be found. One should remind of this circumstance before waving the magic wand of quantisation over an effective Hamiltonian for an inhomogeneous dielectric, be it derived from the quantum electrodynamics of vacuum, or just drawn with some argument from the classical electrodynamics of continua.

## REFERENCES

- [1] E. Schrödinger: *Naturwissenschaften* **23**, (1935) 807.
- [2] F. Miglietta: *Nuovo Cimento D* **20** (1998) 43.
- [3] J. M. Jauch and K. M. Watson: *Phys. Rev.* **74**, (1948) 950, 1485.
- [4] K. Nagy: *Acta Phys. Hung.* **5**, (1955) 95.
- [5] H. Minkowski: *Gött. Nachr., Math.-Phys. Klasse* (1908) 53.
- [6] M. Abraham: *Rend. Circ. Matem. Palermo* **28**, (1909) 1.
- [7] W. Gordon: *Ann. Phys. (Leipzig)* **72**, (1923) 421.
- [8] G. B. Walker and D. G. Lahoz: *Nature* **253**, (1975) 339.
- [9] G. B. Walker, D. G. Lahoz and G. Walker: *Can. J. Phys.* **53**, (1975) 2577.
- [10] G. Marx and K. Nagy: *Acta Phys. Hung.* **4**, (1955) 297.
- [11] W.O. Kermack, W.H. M'Crear and E.T. Whittaker: *Proc. Roy. Soc. Edinburgh* **53**, (1933) 31.
- [12] J.L. Synge: *Quart. J. Math.* **6**, (1935) 199.
- [13] Pham Mau Quan: *Arch. Rat. Mech. and Anal.* **1**, (1957) 54.
- [14] M. Novello and J.M. Salim: *Phys. Rev. D* **63**, (2001) 83511.
- [15] D. Hilbert: *Nachr. Ges. Wiss. Göttingen, Math. Phys. Kl.* 1917, 53.
- [16] M. von Laue: *Phys. Zeits.* **21**, (1920) 659.
- [17] E.T. Whittaker: *Proc. Camb. Phil. Soc.* **24**, (1927) 32.
- [18] C. Lanczos: *Zeits. f. Phys.* **17**, (1923) 32.
- [19] E. J. Post: *Formal Structure of Electromagnetics*. (North-Holland Publishing Company, Amsterdam 1962).
- [20] P. Speziali (Editor): *Albert Einstein-Michele Besso Correspondance, 1903-1955* (Hermann, Paris 1972).
- [21] J. Stachel: *Einstein and the quantum: fifty years of struggle*, in *From quarks to quasars* (University of Pittsburgh Press, Pittsburgh, 1986).

DIPARTIMENTO DI FISICA "A. VOLTA", VIA BASSI 6, PAVIA, ITALY  
*E-mail address:* [Antoci@fisav.unipv.it](mailto:Antoci@fisav.unipv.it)