# Hans-Jürgen Treder and the discovery of confinement in Einstein's unified field theory

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**Abstract**

In the year 1957, when interest in Einstein's unified field theory was fading away for lack of understanding of its physical content, Treder performed a momentous critical analysis of the possible definitions of the electric four-current in the theory. As an outcome of this scrutiny he was able to prove by the E.I.H. method that properly defined point charges, appended at the right-hand side of the field equation $R_{[\underset{\vee}{\mu\nu},\lambda]} = 0$, interact mutually with Coulomb-like forces, provided that a mutual force independent of distance is present too. This unwanted, but unavoidable addition, could not but lay further disbelief on the efforts initiated by Einstein and Schrödinger one decade earlier. However in 1980 Treder himself recalled that the potential $\varphi = a/r + cr$, found by him in 1957, was the one used by particle physicists to account phenomenologically for the spectrum of bound quark systems like mesons. Exact solutions have later confirmed beyond any doubt that Einstein's unified field theory does account in a simple way, already in classical form, for the confinement of pole charges defined by the four-current first availed of by Treder.

In the present paper it is proposed, ad memoriam, a thorough recollection of the article published by Treder in 1957, showing the way kept by him to find what would have been later recognized as confinement in Einstein's unified field theory.

## 1 Introduction

In the year 1957, one decade had elapsed since Einstein had resumed his attempt[1], first formulated already in 1925, to encompass both gravitation and electromagnetism in a generalization of his theory of 1915 based on a nonsymmetric fundamental tensor $g_{\mu\nu}$, and on a nonsymmetric affine connection $\Gamma^{\lambda}_{\mu\nu}$. One decade too had gone by after Schrödinger, by starting from a purely affine approach, had come to announce[2] to have eventually reached "the final affine field laws", that would encompass too both electromagnetism and gravitation in a geometric formulation, very similar to the one proposed by Einstein.

However, despite the intense work done by many relativists and geometers to understand both the mathematical structure and the physical content of what was appropriately called the Einstein-Schrödinger theory, the perspective for this sort of endeavour was not, in 1957, as promising as it had appeared one decade earlier. The formal simplicity of the sets of equations proposed both by Einstein and by Schrödinger had not yet found a counterpart in an equally simple and satisfactory physical interpretation. Already in the 1954/55 report to the Dublin Institute for Advanced Studies, a disappointed Erwin Schrödinger had written: "It is a disconcerting situation that ten years endeavour of competent theorists has not yielded even a plausible glimpse of Coulomb's law."[3].

In the very year 1957 H. Treder, a collaborator of A. Papapetrou in Berlin, published in Annalen der Physik a paper, where a critical scrutiny of the definitions of the electric charge-current density admissible in Einstein's unified field theory is performed[4] for the first time. As a consequence of his analysis of this crucial issue, Treder could show that the negative outcome for the electric force, found both by Infeld[5] and by Callaway[6], by availing of the weak field approximation for solving the equations of motion by the E.I.H. method[7, 8], depended on the choice of the definition of the electric four-current done by those authors. If a different choice is made, allowed for too according to Treder's analysis, the equations of motion stemming from the weak field approximation prove that Einstein's unified field theory does admit of non gravitational forces between charged point particles. But a surprising result is found with Treder's choice of the electric four-current too, for Einstein's theory does not provide in this way a pure Coulomb force between two point charges. A force independent of distance is also present, that cannot be made to vanish by any choice of the constants, because the presence of the latter force is mandatory for the very existence of the Coulomb term.

Already in his paper of 1957, Treder expressed some doubt about the electromagnetic meaning of his finding. Indeed, when the two charged particles are sufficiently far away from each other, the component of the force that is independent of distance will inescapably become the prevailing one, no matter how weak it may be chosen to be.

Scope of the present paper is to reconsider Treder's finding of 1957, based on approximate calculations, in full detail. It will be reminded too that in 1980 Treder himself[9] interpreted his earlier finding as proving that, in the Hermitian version[10] of Einstein's theory, pole point charges of unlike sign, provided by the four-current first considered by him[4], attract mutually with a force independent of distance, hence they are permanently confined entities, like the quarks of chromodynamics are presumed to be.

It will be reminded eventually that exact solutions to the field equations of Einstein's unified field theory belonging to a class found[11] in 1987 confirm with exact arguments [12, 13] the existence of confinement in Einstein's unified field theory.

## 2 Treder's definition of the charge-current in Einstein's unified field theory

It is quite interesting to examine the logical thread followed by Treder in choosing his definition of the "electric" 4-current density. It is evident that in 1957, given the problematic condition of the theory, he feels the need to confront the issue afresh, without being encumbered by prejudices, in particular by the authoritative a priori stipulation, upheld both by Einstein[15] and by Schrödinger[2], according to which, since the new theory did represent the field-theoretical completion of the theory of 1915, no phenomenological sources had to be appended at the right-hand sides of its field equations. For Treder, it is the so-called $+-$ relation that plays a crucial guiding rôle. According to him, it is evident that, since the equation $g_{\underset{+-}{\mu\nu};\lambda} = 0$ provides the definition of the affine connection $\Gamma^{\lambda}_{\mu\nu}$ in terms of the fundamental tensor $g_{\mu\nu}$, it needs to be satisfied everywhere.

As a consequence of this stipulation, also the electromagnetic looking equation $\mathbf{g}^{\underset{\vee}{\mu\nu}}{}_{,\nu} = 0$, that stems[2] from the previous defining equation for $\Gamma^{\lambda}_{\mu\nu}$, needs to be satisfied everywhere. Therefore it is impossible to interpret the latter equation as representing, in Einstein's unified field theory, the first group of Maxwell's equations. In Einstein's theory, $\mathbf{g}^{\underset{\vee}{\sigma\tau}}$ "must be the antisymmetric tensor density dual to the electromagnetic field strength". But this momentous recognition is of scarce help in deciding what 4-vector represents the electric four-current, " because in the unified field theory field strength and induction are not necessarily connected through the relation which we know from Maxwell's theory".

In order to confront the issue, Treder carefully examines, in part **II** of his paper, what suggestions come from the solutions of the weak field, first order approximation of the field equation $R_{[\underset{\vee}{\mu\nu},\lambda]} = 0$, because, "when the unified field theory is expected to have also a macroscopic meaning, it must be required that it allows to describe the existence of pointlike charges in the classical vacuum in the lowest approximation at least, for weak fields". According to Treder, in order to solve the issue of the definition of the electric four-current, there is therefore merit in studying the spherically symmetric, static solution of the weak field approximation of $R_{[\underset{\vee}{\mu\nu},\lambda]} = 0$. Since, $\mathbf{g}^{\underset{\vee}{\mu\nu}}{}_{,\nu} = 0$ must be fulfilled everywhere, the first order approximation of $g_{\underset{\vee}{\mu\nu}}$ must be the dual of the curl of a four-vector:

$$\underset{1}{g}_{\underset{\vee}{\mu\nu}} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\tau}\left(\varphi_{\underline{\tau},\underline{\sigma}} - \varphi_{\underline{\sigma},\underline{\tau}}\right),$$

and in the Lorentz gauge $\underset{1}{R}_{[\underset{\vee}{\mu\nu},\lambda]} = 0$ specialises to

$$\Box^2\,\varphi^\sigma = 0.$$

In the static case $\varphi^\lambda = (0,0,0,\varphi)$, and the latter equation specialises further to

$$\Delta\Delta\varphi = 0,$$

whose general, spherically symmetric solution reads[1]

$$\varphi = \frac{a}{r} + b + cr + dr^2.$$

After dropping the term $dr^2$, that leads to the divergent behaviour of $\underset{1}{g}_{\mu\nu}$ for $r = \infty$, and the unessential constant $b$, $\varphi$ takes the paradigmatic form

$$\varphi = \frac{a}{r} + cr.$$

Treder then looks for the charge density definitions that are compatible with this form of $\varphi$ in the previously specified sense, namely, he looks for the $\delta$ functions that can be generated through either single or double application

[1] Biharmonic equations are to be expected in theories with quadratic Lagrangians. It is remarkable that the same weak field expression of a potential is found too, under suitable conditions, in the framework of Poincaré gauge field theory. For instance, with the purpose and interpretation of confinement this was considered in the paper "Short-range confining component in a quadratic Poincaré gauge theory of gravitation"[14].

of the Laplace operator $\Delta$ to $\varphi$, and seeks to what exact charge-current density definitions they shall correspond as particular approximate, weak field cases. He notices that by applying the Laplace operator once to the first term of $\varphi$ one gets

$$\Delta\left(\frac{a}{r}\right) = -4\pi a\delta(\mathbf{r})$$

i.e. a $\delta$ source that is a particular static, first order approximation of the source term $s_{\mu\nu\lambda}$ occurring in the general four-current definition

$$g_{[\underset{\vee}{\mu\nu},\,\lambda]} \equiv -s_{\mu\nu\lambda}.$$

Against this option, however, Treder raises the objection that it does not allow for a free choice of the charge-current density, like it happens instead in Maxwell's theory, because, due to the field equation $R_{[\underset{\vee}{\mu\nu},\,\lambda]} = 0$, the charge-current density defined in this way is constrained to fulfill, in the weak field approximation, a differential, d'Alembert equation:

$$\Box \underset{1}{s}_{\mu\nu\lambda} = 0.$$

By applying twice the Laplace operator to the second term of $\varphi$ one gets again a $\delta$ function:

$$\Delta\Delta(cr) = -8\pi c\delta(\mathbf{r}).$$

Treder notices that this particular $\delta$ charge density is a weak field, static instance of a charge-current density $s_{\mu\nu\lambda}$ defined by

$$R_{[\underset{\vee}{\mu\nu},\,\lambda]} \equiv \frac{1}{2}s_{\mu\nu\lambda},$$

i.e. of a charge-current density appended in a phenomenological way at the right-hand side of the field equation $R_{[\underset{\vee}{\mu\nu},\,\lambda]} = 0$. As such, this four-current density can be assigned at will (subject to the conservation law) like it occurs in Maxwell's theory. When both terms of $\varphi$ are considered, the double application of the Laplacian leads instead to the point source expression

$$\Delta\Delta\varphi = -4\pi\left(a\Delta\delta(\mathbf{r}) + 2c\delta(\mathbf{r})\right).$$

This is the structure of each of the $n$ point charges that Treder introduces in his derivation of the equations of motion in the weak field, first order approximation of the E.I.H. method performed in **III**, after having assumed that, for any charge I, the ratio $c_{\mathrm{I}}/a_{\mathrm{I}}$ is a universal constant $\tau$.

## 3 Treder's discovery of confinement in Einstein's unified field theory

After the momentous assumptions done in **II**, deriving in **III**, by the E.I.H. method, the equations of motion for $n$ charged particles in the weak field, slow motion approximation is for Treder a straightforward, routine move, done in the footsteps of Infeld[5]. It leads however to a highly perplexing end result. Like Treder, let us consider for simplicity the case $n = 2$, when the static potential $\varphi$ comes to read

$$\varphi = \varphi_{\mathrm{I}} + \varphi_{\mathrm{II}} = \frac{a_{\mathrm{I}}}{r_{\mathrm{I}}} + c_{\mathrm{I}} r_{\mathrm{I}} + \frac{a_{\mathrm{II}}}{r_{\mathrm{II}}} + c_{\mathrm{II}} r_{\mathrm{II}}$$

and, to the required order of approximation, although with some inappropriateness in the language, one may assert that the Cartesian components of the "electric" force that the field of a pointlike charge II exerts on a pointlike charged particle I is given by

$$\underset{4}{\overset{\mathrm{I}}{\mathcal{L}}}_i \equiv \frac{1}{4\pi} \oint_{\mathrm{I}}^{0} 2 \underset{4}{L}_{ik} \overset{\mathrm{I}}{n}{}^k \, \mathrm{d} \overset{\mathrm{I}}{S}$$
$$= 2 c_{\mathrm{I}} a_{\mathrm{II}} \frac{\xi_i}{\varrho^3} + 2 a_{\mathrm{I}} c_{\mathrm{II}} \frac{\xi_i}{\varrho^3} - 2 c_{\mathrm{I}} c_{\mathrm{II}} \frac{\xi_i}{\varrho}$$
$$\xi^i = x^i_{\mathrm{I}} - x^i_{\mathrm{II}} \text{ and } \varrho^2 = \xi^s \xi^s,$$

when the integral is extended to a closed surface surrounding only particle I. When this expression, found by Treder as a direct outcome of his pondered choice of the definition of the four-current in Einstein's unified field theory, appeared in print[4], it was not new. It had been written already by V. V. Narlikar and B. R. Rao in their paper of 1956, entitled "The equations of motion of particles in the unified field theory of Einstein (1953)"[16]. However we feel entitled to attribute only to Treder the correct interpretation of this surprising result, and to continue its analysis along the line drawn in his article of 1957, because the interpretation considered by Narlikar and Rao is instead based on an untenable assumption. For these authors, the four-current responsible for the above written force is proportional to $g_{\underset{\vee}{[\mu\nu},\lambda]}$, hence the corresponding charges are by no means pointlike, but diffused in the whole space and overlapping. It is obvious that point particles are instead needed to make sense of an E.I.H. calculation.

However, although Treder's choice of the four-current leads in the present case to pointlike charges, i.e. his E.I.H. calculation is conceptually faultless, the “electric” interpretation of the resulting force soon arose perplexity. F.A.E. Pirani, when commenting[17] Treder’s paper for the “Mathematical Reviews”, wrote:

> The author proposes a new definition of charge-current in Einstein’s “weak” non-symmetric unified theory [The meaning of relativity, 3rd ed., revised, Princeton, 1950, Appendix II]. In the lowest approximation he obtains the Coulomb force between point charges, but also, unfortunately, an additional force independent of distance.

As noted by Treder, only if one makes the additional assumption that the ratio $c_{\rm I}/a_{\rm I},\ i=1,\cdots,\ n$ is a universal constant $\tau$ does one get a law of force that approximates the ordinary Coulomb law, as long as the inequality

$$\tau \ll \frac{1}{\varrho^2}$$

is satisfied. But of course, the force independent of distance cannot be hidden out: it will always become the prevailing one when the charged particles are posited farther and farther away from each other. Therefore, the “electric” interpretation of the result, although it found its adherents, e.g. in[18, 19], was never considered to be a satisfactory one, not even by Treder himself at the very moment of its finding, as it transpires from the concluding remarks in **V**.

It is evident that in 1957 a force independent of distance between point charges could not be thought to be of much use in theoretical physics. Therefore the very existence of such a force in Einstein’s unified field theory, so keenly brought into evidence by Treder, could not but help laying further discredit on the theoretical endeavour inaugurated by Einstein and Schrödinger one decade earlier.

However, what ideas are of interest to theoreticians change with the lapse of time and, as mentioned in the Introduction, in 1980 Treder[9] might well wonder whether his early finding could not be reinterpreted as the evidence that Einstein’s theory allows, already in classical form, for the confinement of quarks, i.e. it can account for both the strong and the gravitational force in a unified way. Phenomenological potential models introduced at the time[20, 21] used in fact a linear combination of a Coulomb and of a linear radial potential, just like the one found by Treder in 1957, to account satisfactorily for the spectroscopy of hadrons. But, one should ask: if Einstein’s theory allows for a unified description of both the strong and the gravitational interaction, where must one look for electromagnetism in the theory? What entity represents, in the theory, the long sought for electric four-current?

## 4 What the exact solutions have to say

After the finding, in 1987, of a class of exact solutions of the Einstein-Schrödinger equations intrinsically depending on three coordinates[11] it was noticed, from the study of particular solutions, that perhaps Treder's injunction, that both the equations $g_{\underset{+-}{\mu\nu};\lambda} = 0$ and $\mathbf{g}^{\underset{\vee}{\mu\nu}}{}_{,\nu} = 0$ have to be satisfied everywhere, is too restrictive, thereby leading to a loss of valuable physical content of the theory. In 1978 Borchsenius[22] had shown how to obtain that a phenomenological four-current may appear at the right-hand sides of the two equations just mentioned above without destroying the invariance of the theory under transposition. Therefore, in the footsteps of the successful phenomenological completion of the general relativity of 1915, the way was open for interpreting Einstein's theory of the nonsymmetric field as a theory admitting both a symmetric energy tensor $T_{\mu\nu}$ and two distinct, conserved four-currents $j^\varrho$ and $K_{\mu\nu\lambda}$ like phenomenological sources[23]. Its field equations, that reduce to the original ones wherever sources are absent, then read:

$$\mathbf{g}^{\mu\nu}_{\ ,\lambda} + \mathbf{g}^{\sigma\nu}\Gamma^{\mu}_{\sigma\lambda} + \mathbf{g}^{\mu\sigma}\Gamma^{\nu}_{\lambda\sigma} - \mathbf{g}^{\mu\nu}\Gamma^{\tau}_{\underline{\lambda\tau}} = \frac{4\pi}{3}(\mathbf{j}^{\mu}\delta^{\nu}_{\lambda} - \mathbf{j}^{\nu}\delta^{\mu}_{\lambda}),$$

$$\mathbf{g}^{\underset{\vee}{\varrho\sigma}}{}_{,\sigma} = 4\pi\mathbf{j}^{\varrho},$$

$$\bar{R}_{\underline{\mu\nu}}(\Gamma) = 8\pi(T_{\mu\nu} - \frac{1}{2}s_{\mu\nu}s^{\varrho\sigma}T_{\varrho\sigma}),$$

$$\bar{R}_{[\underset{\vee}{\mu\nu},\lambda]} = 8\pi K_{\mu\nu\lambda},$$

where, like in Treder's paper[4], $s_{\mu\nu}$ is the metric tensor defined by Kurşunoğlu[24] and Hély[25] as

$$s^{\mu\nu} = \sqrt{\frac{g}{s}}g^{\underline{\mu\nu}}, \quad s_{\sigma\tau}s^{\mu\tau} = \delta^{\mu}_{\sigma}.$$

$\bar{R}_{\mu\nu}$ is the symmetrised Ricci tensor of Borchsenius:

$$\bar{R}_{\mu\nu}(\Gamma) = \Gamma^{\varrho}_{\mu\nu,\varrho} - \frac{1}{2}\left(\Gamma^{\varrho}_{\mu\varrho,\nu} + \Gamma^{\varrho}_{\nu\varrho,\mu}\right) - \Gamma^{\alpha}_{\mu\varrho}\Gamma^{\varrho}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\varrho}_{\alpha\varrho},$$

that reduces to the plain one wherever $\mathbf{j}^\varrho$ vanishes. With these definitions, and with the semicolon ";" standing for the covariant differentiation performed with the Christoffel symbols built with $s_{\mu\nu}$, the contracted Bianchi identities of the theory come to read

$$\mathbf{T}^{\lambda\sigma}_{\ ;\sigma} = \frac{1}{2}s^{\lambda\varrho}\left(\mathbf{j}^{\tau}\bar{R}_{\underset{\vee}{\varrho\tau}}(\Gamma) + K_{\tau\varrho\sigma}\mathbf{g}^{\underset{\vee}{\sigma\tau}}\right), \quad \mathbf{T}^{\mu\nu} = \sqrt{-s}s^{\mu\varrho}s^{\nu\sigma}T_{\varrho\sigma},$$

a perspicuous enough writing.

To the previously mentioned class of solutions belongs a particular exact solution that is static and endowed with pole charges built with the current $K_{\tau\varrho\sigma}$. Its details are given elsewhere[12, 13] and will not be repeated here. Suffice it to say that the solution confirms beyond any possible doubt what the approximate result found by Treder in 1957 already said, i.e. that Einstein's unified field theory, when complemented with the phenomenological four-current $K_{\tau\varrho\sigma}$, allows describing point charges interacting mutually with forces independent of distance. In the Hermitian version of the theory two charges of unlike sign mutually attract, hence are permanently confined entities. As far as exact solutions are concerned, the theory therefore provides examples both of gravitating bodies[26] and of bodies interacting like quarks are expected to do.

But to the same class belongs another exact solution[27], that is static too, and whose field $\mathbf{g}^{\mu\nu}_{\vee}$ is associated with charge density built with the other four-current, $j^{\varrho}$. Since, outside the charges, the field fulfils the field equation $\mathbf{g}^{\mu\nu}_{\vee}{}_{,\nu} = 0$, while the unsolicited equation

$$g_{[\underset{\vee}{\mu\nu},\lambda]} = 0$$

is satisfied everywhere, one cannot help recognizing in this solution the general electrostatic solution of Einstein's unified field theory. Moreover if, in the adopted representative space, one puts the charge distribution on $n$ localized, closed two-surfaces, it is possible[27] to generate, in the metric sense, the charge distribution of $n$ pointlike, spherically symmetric charges. This occurrence only happens when the charges occupy mutual positions that correspond, with all the accuracy needed to meet with the most stringent empirical results, to the mutual positions dictated by Coulomb's law for the equilibrium condition of $n$ pointlike charges.

As far as the evidence associated with a particular exact solution can go, this result constitutes a partial, but hopefully enlightening answer to the two questions raised at the end of the previous section, about the presence of electromagnetism in Einstein's theory, and about the identification of the electric four-current.

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