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A forgotten argument by Gordon uniquely selects Abraham's tensor as the energy-momentum tensor for the electromagnetic field in homogeneous, isotropic matter.

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Riassunto. - Vista la situazione attuale del problema del tensore elettromagnetico dell'energia nella materia è forse utile rammentare un argomento dimenticato, dato nel 1923 da W. Gordon. Per un mezzo materiale che a riposo sia omogeneo ed isotropo tale argomento permette di ricondurre il problema suddetto a un problema analogo, definito nel vuoto della relatività generale (in presenza di una metrica effettiva γ_{ik} opportunamente determinata). Per questo secondo problema la forma della lagrangiana elettromagnetica è nota, e la determinazione del tensore dell'energia si compie subito, perchè basta eseguire la derivazione della lagrangiana così scelta rispetto alla metrica vera g_{ik}. Si seleziona allora il tensore di Abraham come tensore elettromagnetico dell'energia per un mezzo che a riposo sia omogeneo ed isotropo.

Summary. - Given the present status of the problem of the electromagnetic energy tensor in matter, there is perhaps use in recalling a forgotten argument given in 1923 by W. Gordon. Let us consider a material medium which is homogeneous and isotropic when observed in its rest frame. For such a medium, Gordon's argument allows to reduce the above mentioned problem to an analogous one, defined in a general relativistic vacuum (*i.e.* in presence of a suitably determined metric γ_{ik}). For the latter problem the form of the Lagrangian is known already, hence the determination of the energy tensor is a straightforward matter. One just performs the Hamiltonian derivative of the Lagrangian chosen in this way with respect to the true metric g_{ik}. Abraham's tensor is thus selected as the electromagnetic energy tensor for a medium which is homogeneous and isotropic in its rest frame.

P.A.C.S.: **03.50-z** - Classical field theory.
P.A.C.S.: **04.20-q** - Classical general relativity.

1. Introduction.

The reader of old scientific literature is sometimes confronted with findings that somewhat shake his belief that physics, at variance with other sorts of human endeavours, undergo a more or less steady progress, maybe interlarded from time to time with Kuhn-like revolutions [1], but always driven by rational motivations. He is in fact forced to recognize the existence of forgotten papers, whose indisputable relevance for some reason went unnoticed at the time of their publication. Due to some "paradigm shift" and to the associated, ingrained habit of physicists to occupy their minds mainly with state of the art contributions, it is quite difficult that such papers may resurface from the oblivion in which their were cast at the very moment of their appearance.

As far as we could ascertain, this was the destiny of the article "Zur Lichtfortpflanzung nach der Relativitätstheorie" published in 1923 by W. Gordon [2]. It is true that this paper was published just before the major paradigm shift that placed quantum mechanics at center stage and relegated general relativistic ideas to a somewhat lesser rôle. It is also true that in 1960 J.L. Synge [3] was well aware of the virtues, for the geometrical optics of non-dispersive media, of the effective metric introduced by Gordon in the above mentioned paper. In that book Synge recalls that, in the limit of geometrical optics, the rays are null geodesics with respect to Gordon's effective metric, but he does not remember that the original finding of this property (although only for homogeneous, isotropic media) is due to Gordon. About this finding he mentions in fact only later authors [4],[5].

However, the main reason why Gordon's paper should have been considered at the time of its appearance and remembered thereafter is, in our opinion, another one and, to our knowledge, it has remained completely unnoticed by everybody, if one excludes the attentive reviewer of Gordon's article for "Physikalische Berichte" [6]: that paper contains a clear-cut argument for selecting, through a clever *reductio ad vacuum*, the stress-energy-momentum tensor of the electromagnetic field in non-dispersive matter, provided that the latter be homogeneous and isotropic when considered in its rest frame.

We have found no track of Gordon's argument in the huge literature that has been produced on this controversial issue in the decades elapsed since then; we feel therefore enticed into recalling it here. We do so not for the historical record, but for the very reason that the problem of the energy tensor of the macroscopic electromagnetic field in matter, after so many efforts both on the theoretical and on the experimental side, and after the above mentioned paradigm shift, is still with us, as open and unsolved as it was at the time when Minkowski [7] and Abraham [8] provided their well known, diverging answers.

2. Field equations and constitutive relations for electromagnetism in matter and in vacuo.

In order to best appreciate the general relativistic argument conceived by Gordon one should remind of the position occupied by Maxwell's equations in the structure of space-time.

Remarkably enough, neither a metric nor an affine connection are required for their definition; these equations can be written as soon as the concept of four-dimensional differentiable manifold is introduced [9]. In such a manifold one can consider a contravariant skew tensor density [1] $\mathbf{H}^{ik}$, a covariant skew tensor F_{ik}, and write the "naturally invariant" equations [2]

$$\mathbf{H}^{ik}_{,k} = \mathbf{s}^i, \tag{1}$$

and

$$F_{[ik,m]} = 0. \tag{2}$$

The vector density $\mathbf{s}^i$, defined by the left-hand side of eq. (1), is assumed to represent the electric four-current density, while the comma signals ordinary differentiation, and we have set $F_{[ik,m]} \equiv \frac{1}{3}(F_{ik,m} + F_{km,i} + F_{mi,k})$. Equations (1) and (2) express Maxwell's equations in general curvilinear co-ordinates; they constitute two sets of four equations, which fulfil two identities:

[1] Boldface letters are henceforth used to indicate densities.

[2] For ease of comparison, we conform to the notation and to the conventions adopted by Gordon in Ref. [2].

$$\mathbf{H}^{ik}_{,k,i} = \mathbf{s}^i_{,i} = 0, \tag{3}$$

that accounts for the conservation of the electric four-current, and

$$\mathbf{e}^{ikmn} F_{[ik,m],n} = 0, \tag{4}$$

where $\mathbf{e}^{ikmn}$ is the totally antisymmetric symbol of Ricci and Levi Civita. Therefore equations (1) and (2) cannot be used for determining, in a given co-ordinate system, both $\mathbf{H}^{ik}$ and F_{ik}, which possess together 12 independent components. Equations (1) and (2) need to be complemented with the constitutive relations of electromagnetism, *i.e.* with a set of six tensor equations of some sort that allow to uniquely define *e.g.* $\mathbf{H}^{ik}$ in terms of F_{ik} and of whatever additional fields may be needed for specifying the properties of the electromagnetic medium that one is considering. In this way the number of independent components of $\mathbf{H}^{ik}$ and F_{ik} is reduced to 6, and eqs. (1) and (2) may suffice for predicting the evolution of the electromagnetic field (for a given four-current density $\mathbf{s}^i$).

When the dependence of $\mathbf{H}^{ik}$ on F_{ik} is assumed to be algebraic and linear, the constitutive relations can read [10]

$$\mathbf{H}^{ik} = \tfrac{1}{2}\mathbf{X}^{ikmn} F_{mn}, \tag{5}$$

and the electromagnetic properties of the medium are summarized by the four-index tensor density $\mathbf{X}^{ikmn}$, which is skew both in the first and in the second pair of indices. A particular example of the constitutive relations (5) is provided by the very peculiar medium that we call vacuum. In this case one writes:

$$\mathbf{X}^{ikmn}_{vac.} = \sqrt{g}(g^{im}g^{kn} - g^{in}g^{km}), \tag{6}$$

i.e. the constitutive relations entail the metric tensor g_{ik} and $g \equiv -det(g_{ik})$ in the way well known from general relativity. For the vacuum case Maxwell's equations are more usually written as:

$$\mathbf{F}^{ik}_{,k} = \mathbf{s}^i, \tag{7}$$

$$F_{[ik,m]} = 0, \tag{8}$$

in terms of a skew tensor F_{ik} and of the associated contravariant tensor density $\mathbf{F}^{ik} \equiv \sqrt{g} g^{im} g^{kn} F_{mn}$.

3. The derivation of the electromagnetic field equations and of the energy tensor for the vacuum of general relativity.

It is well known [11] that, if one defines the electromagnetic field in terms of the four-potential φ_i:

$$F_{ik} = \varphi_{k,i} - \varphi_{i,k}, \tag{9}$$

then the homogeneous equations (8) are *a priori* satisfied, and the inhomogeneous equations (7) can be derived through the Hamilton principle, by starting from the Lagrangian density

$$\mathbf{L} = \tfrac{1}{4}\mathbf{F}^{ik} F_{ik} - \mathbf{s}^i \varphi_i. \tag{10}$$

It is less known that the lame variational method described above can be dropped, that *both* sets of Maxwell equations can be derived through the Hamilton principle from the Lagrangian density reported above [12]. In fact, there is no need to restrict *a priori* F_{ik} to be the curl of a four-vector φ_i. One can instead start with a general skew tensor F_{ik}, that can always be written as the sum of the curl of a potential and of the dual to the curl of an "antipotential":

$$F_{ik} = \varphi_{k,i} - \varphi_{i,k} + e_{ik}{}^{mn}(\psi_{n,m} - \psi_{m,n}). \tag{11}$$

Here $e_{ik}{}^{mn}$ is the tensor obtained from the totally antisymmetric tensor density $\mathbf{e}^{ikmn}$ in the usual way. By asking that the variations of $A = \int \mathbf{L}\, dS$ ($dS = dx^1\, dx^2\, dx^3\, dx^4$) with respect to φ_i and to ψ_i separately vanish one immediately obtains eqs. (7) and (8).

The well known form of the energy tensor density $\mathbf{T}_{ik}$ for the electromagnetic field *in vacuo* is eventually obtained through the general method [11] inaugurated by Hilbert, i.e. by performing the variation of the Lagrangian density (10) with respect to the metric tensor g^{ik}:

$$\mathbf{T}_{ik} \equiv 2\frac{\delta \mathbf{L}}{\delta g^{ik}} = \mathbf{F}_i{}^n F_{kn} - \tfrac{1}{4} g_{ik} \mathbf{F}^{mn} F_{mn}. \tag{12}$$

If Finzi's variational method is adopted, the derivation of the energy tensor must be performed by keeping into account the just obtained homogeneous field equations, which dictate that F_{ik} is the curl of a four-vector φ_i and does not contain the metric.

4. The constitutive relations of electromagnetism for an isotropic, homogeneous medium in general relativity.

The constitutive relations of electromagnetism for a linear, non-dispersive medium will have in general the form of eq. (5). We notice that, since $\mathbf{X}^{ikmn}$ is skew both in the first and in the second pair of indices, its components can be given through a 6×6 matrix.[3] We shall henceforth deal with the case of a medium which is homogeneous and isotropic, when considered at rest. In order to recognize such a medium in general relativity we first transform $\mathbf{X}^{ikmn}$ to a co-ordinate system for which, at a given event, $g_{ik} = \eta_{ik} \equiv diag(1, 1, 1, -1)$. In the new co-ordinate system $\mathbf{X}^{ikmn}$ will in general display off-diagonal components. We can now perform a Lorentz transformation, that will not change the form of the metric at the event under question. Suppose that after the Lorentz transformation $\mathbf{X}^{ikmn}$ (in the matrix rendering) becomes diagonal, and that the first three components on the diagonal are equal to, say, $-\epsilon$, while the remaining three read $1/\mu$. If this is the case we have to do (at the considered event) with an isotropic medium of dielectric constant ϵ and of magnetic permeability μ. If we apply the same procedure at any event, and we find that the matrix can always be put in diagonal form with the same values for the components as before, the medium is also homogeneous.

The constitutive relations for an isotropic, homogeneous medium can be written in a simple form, that is due to Minkowski [7], and can be extended without change to general relativity. Let

$$u^i = \frac{dx^i}{\sqrt{-ds^2}} \tag{13}$$

be the four-velocity of matter, for which $u_i u^i = -1$. One defines the four-vectors

$$F_i = F_{ik} u^k, \quad H_i = H_{ik} u^k, \tag{14}$$

where $H_{ik} \equiv (1/\sqrt{g}) g_{ip} g_{kq} \mathbf{H}^{pq}$ is the covariant tensor associated with $\mathbf{H}^{ik}$. Then the above mentioned constitutive relations simply read

$$H_i = \epsilon F_i, \tag{15}$$

$$u_i F_{km} + u_k F_{mi} + u_m F_{ik} = \mu\big(u_i H_{km} + u_k H_{mi} + u_m H_{ik}\big). \tag{16}$$

As shown by Gordon [2], these eight equations, that entail two identities, are equivalent to the six equations:

$$\mu H^{ik} = F^{ik} + (\epsilon\mu - 1)(u^i F^k - u^k F^i) \tag{17}$$

that can be easily cast into the form of eq. (5). But the right-hand side of eq. (17) can be rewritten as

[3] The correspondence between the matrix indices and the pairs of skew tensor indices is assumed to be as follows: $1 \Leftrightarrow 41,\ 2 \Leftrightarrow 42,\ 3 \Leftrightarrow 43,\ 4 \Leftrightarrow 23,\ 5 \Leftrightarrow 31,\ 6 \Leftrightarrow 12.$

$$F_{rs}\{g^{ir}g^{ks}-(\epsilon\mu-1)\big(u^iu^rg^{ks}+u^ku^sg^{ir}\big)\}.$$

We can freely add the term $(\epsilon\mu-1)^2u^iu^ku^ru^s$ within the curly brackets, since it will give no contribution, due to the antisymmetry of F_{rs}. Then eq. (17) comes to read

$$\mu H^{ik}=\big(g^{ir}-(\epsilon\mu-1)u^iu^r\big)\big(g^{ks}-(\epsilon\mu-1)u^ku^s\big)F_{rs}. \tag{18}$$

Let us define the "effective metric tensor"

$$\gamma^{ik}=g^{ik}-(\epsilon\mu-1)u^iu^k, \tag{19}$$

whose inverse is

$$\gamma_{ik}=g_{ik}+\big(1-\frac{1}{\epsilon\mu}\big)u_iu_k; \tag{20}$$

then we can bring eq. (17) to the form:

$$\mu\mathbf{H}^{ik}=\sqrt{g}\gamma^{ir}\gamma^{ks}F_{rs}. \tag{21}$$

Since $g\equiv -det(g_{ik})$, we shall pose $\gamma\equiv -det(\gamma_{ik})$. Then the ratio γ/g shall be an invariant. Its calculation can be performed in the co-ordinate system in which $u^1=u^2=u^3=0$, and one finds

$$\gamma=\frac{g}{\epsilon\mu},$$

so that eq. (21) can be rewritten as

$$\mathbf{H}^{ik}=\sqrt{\frac{\epsilon}{\mu}}\sqrt{\gamma}\gamma^{ir}\gamma^{ks}F_{rs}, \tag{22}$$

which, apart from the constant factor $\sqrt{\epsilon/\mu}$, is just the constitutive relation of the general relativistic vacuum for which γ_{ik} acts as metric.

We shall henceforth enclose in round brackets the indices which are either moved with γ^{ik} and γ_{ik}, or generated by performing the Hamiltonian derivative with respect to the latter tensors; therefore eq. (22) will be rewritten as

$$\mathbf{H}^{ik}=\sqrt{\frac{\epsilon}{\mu}}\sqrt{\gamma}F^{(i)(k)}. \tag{23}$$

5. The Lagrangian for the electromagnetic field in an isotropic, homogeneous medium and the energy tensor derived from it.

If the metric field is γ_{ik}, according to eq. (10) the Lagrangian density for the electromagnetic field *in vacuo* reads:

$$\mathbf{L}=\tfrac{1}{4}\sqrt{\gamma}F^{(i)(k)}F_{ik}-\mathbf{s}^i\varphi_i. \tag{10'}$$

where F_{ik} will be given either by eq. (9) or by

$$F_{ik}=\varphi_{k,i}-\varphi_{i,k}+\frac{1}{\sqrt{\gamma}}e_{(i)(k)}{}^{mn}(\psi_{n,m}-\psi_{m,n}). \tag{11'}$$

in compliance with the complete variational method proposed by Finzi. Selecting the Lagrangian density for the electromagnetic field in the medium under question is then reduced to a straightforward matter. Gordon [2] writes:

$$\mathbf{L}' = \frac{1}{4}\sqrt{\frac{\epsilon}{\mu}}\sqrt{\gamma}F^{(i)(k)}F_{ik} - \mathbf{s}^i\varphi_i, \tag{24}$$

where F_{ik} can be presently defined by eq. (11'). Equating to zero the independent variations of the action $\int \mathbf{L}'\, dS$ with respect to φ_i and to ψ_i will produce the Maxwell's equations (1) and (2) respectively, *a priori* complemented by the constitutive relations (15) and (16).

It is now easy to derive the energy tensor for the electromagnetic field by starting from the derivation that one performs *in vacuo*, when the metric field γ_{ik} is present. In that case, in keeping with the general definition (12), one writes:

$$\delta\mathbf{L} \equiv \frac{1}{2}\mathbf{T}_{(i)(k)}\delta\gamma^{ik}, \tag{12'}$$

hence one gets

$$\mathbf{T}_{(i)}^{\ \ (k)} = \sqrt{\gamma}\big(F_{ir}F^{(k)(r)} - \frac{1}{4}\delta_i^{\ k}F_{rs}F^{(r)(s)}\big), \tag{25}$$

as already shown in § 3. When the matter under question is present one instead writes:

$$\delta\mathbf{L}' \equiv \frac{1}{2}\mathbf{T}'_{(i)(k)}\delta\gamma^{ik}, \tag{26}$$

where $\mathbf{L}'$ is given by eq. (24), and finds

$$\mathbf{T}'^{\ (k)}_{(i)} = F_{ir}\mathbf{H}^{kr} - \frac{1}{4}\delta_i^{\ k}F_{rs}\mathbf{H}^{rs}, \tag{27}$$

which is just the general relativistic version of the form proposed by Minkowski in his fundamental work [7]. We shall drop henceforth the prime in the expression of the energy tensor, since its omission will not lead to confusion.

$\mathbf{T}_{(i)}^{\ \ (k)}$, however, cannot be the energy tensor density that we are seeking, because it is defined with respect to the effective metric γ_{ik}, not with respect to the true metric g_{ik}, the only one that accounts for the structure of space-time and, via the Einstein tensor, for its stress-energy-momentum tensor T_{ik}.

In order to find the relation between $\mathbf{T}_{ik}$ and $\mathbf{T}_{(i)(k)}$ we simply need to express $\delta\gamma^{ik}$ in terms of δg^{ik}. From eq. (13) one derives the variation of u^i induced by the variation δg^{mn} of the metric:

$$\delta\, u^i = \frac{1}{2}u^i u^m u^n \delta g_{mn} = -\frac{1}{2}u^i u_m u_n \delta g^{mn}, \tag{28}$$

hence from the definition (19) we obtain:

$$\delta\,\gamma^{ik} = \delta\big\{g^{ik} - (\epsilon\mu - 1)u^i u^k\big\} = \delta g^{ik} + (\epsilon\mu - 1)u^i u^k u_m u_n \delta g^{mn}. \tag{29}$$

Therefore, since $\mathbf{T}_{ik}\delta g^{ik} = \mathbf{T}_{(i)(k)}\delta\gamma^{ik}$, we get immediately:

$$\mathbf{T}_{\,ik} = \mathbf{T}_{(i)(k)} + (\epsilon\mu - 1)u_i u_k \mathbf{T}_{(m)(n)}u^m u^n. \tag{30}$$

The mixed components of $\mathbf{T}_{ik}$ can be obtained by multiplying the left-hand side and the second term at the right-hand side of eq. (30) by g^{kq}, while the first term at the right-hand side is multiplied by $\gamma^{kq} + (\epsilon\mu - 1)u^k u^q$. In this way we find

$$\mathbf{T}_i^{\ q} = \mathbf{T}_{(i)}^{\ \ (q)} + (\epsilon\mu - 1)\big\{\mathbf{T}_{(i)(k)}u^k + u_i\mathbf{T}_{(m)(n)}u^m u^n\big\}u^q. \tag{31}$$

In the co-ordinate system for which at a given event $g_{ik} = \eta_{ik}$ and $u^1 = u^2 = u^3 = 0,\ u^4 = 1$, the covariant four-vector within the curly brackets has the components $\mathbf{T}_{(\alpha)(4)},\ 0\ (\alpha = 1, 2, 3)$. But under the circumstances chosen above eq. (30) says that $\mathbf{T}_{(\alpha)(4)} = \mathbf{T}_{\alpha 4}$, hence in a general co-ordinate system one can write:

$$\mathbf{T}_i^{\ q} = \mathbf{T}_{(i)}^{\ \ (q)} + (\epsilon\mu - 1)\big\{\mathbf{T}_{ik}u^k + u_i\mathbf{T}_{mn}u^m u^n\big\}u^q, \tag{32}$$

i.e., according to eq. (27)

$$T_i^{\ k} = F_{ir}H^{kr} - \tfrac{1}{4}\delta_i^{\ k}F_{rs}H^{rs} - (\epsilon\mu - 1)\Omega_i u^k, \tag{33}$$

where Minkowski's "Ruh-Strahl" [7]

$$\Omega^i = -\big(T_k^{\ i}u^k + u^i T_{mn}u^m u^n\big) \tag{34}$$

has been introduced. Since $\Omega^i u_i = 0$, by substituting (33) into (34) one finds

$$\Omega^{\,i} = F_m H^{im} - F_m H^m u^i = u_k F_m\big(H^{ik}u^m + H^{km}u^i + H^{mi}u^k\big). \tag{35}$$

Equations (33) and (35) define the extension to general relativity of the energy tensor proposed by Abraham [8] for the electrognetic field in an isotropic, homogeneous medium.

6. Other arguments.

The weight of Gordon's argument is better appreciated if one considers also other proposals that have been done for selecting the energy tensor of the electromagnetic field in matter.

Among them a relevant position is occupied by the argument originally outlined by Scheye [13], fully developed by v. Laue [14], and revisited by Møller [15]. We have already mentioned in the Introduction that in a homogeneous, isotropic medium the light rays, in the limit of geometrical optics, are null geodesics [2] with respect to the effective metric γ_{ik} given by eq. (20). This result was originally found by Gordon, and subsequently extended to a non-dispersive, isotropic medium [3]. The derivation of this result only entails Maxwell's equations and the constitutive relations; no knowledge of the energy-momentum tensor is required for obtaining it. The ray four-velocity [2], [3] turns out to be a timelike four-vector with respect to the true metric g_{ik}, *i.e.* the ray can accompany a material particle in its path, and keep it permanently illuminated.

In the celebrated paper "Zur Minkowskischen Elektrodynamik der bewegten Körper" [14] v. Laue considers a plane electromagnetic wave propagating in a medium whose constitutive relations are given by eqs. (15) and (16). For that wave he introduces a (three-vector) velocity of radiation v_α ($\alpha = 1,2,3$), given by the ratio between the three-vector $cT_{4\alpha}$, that defines the energy current density of the electromagnetic field, and its energy density T_{44}:

$$v_\alpha = \frac{cT_{4\alpha}}{T_{44}}, \tag{36}$$

where c is the speed of light in the vacuum of special relativity. Laue then postulates the following selecting criterion for the energy tensor of the electromagnetic field: T_{ik} must be such that the timelike four-vector

$$w_i = \left(\frac{v_\alpha}{\sqrt{c^2 - v^2}}\ ,\ \frac{c}{\sqrt{c^2 - v^2}}\right) \tag{37}$$

exist and coincide with the four-velocity of the ray that directly stems from Maxwell's equations. Laue's criterion excludes Abraham's tensor from the permitted ones, while it allows all the energy tensors whose fourth row is proportional to the fourth row of Minkowski's tensor.

How seriously shall we take Laue's criterion? It depends on one's degree of conviction about the "physical reality", *i.e.*, for the ingrained positivist, about the observability of the energy density and of the energy current density of the electromagnetic field. One cannot forget the impressive examples provided by Bopp [16] for the odd behaviour exhibited by the Poynting vector already *in vacuo*.[4)] Nor one can help sharing

[4)] For instance, consider two electromagnetic plane waves whose wavevectors lay in the same plane, and make a certain angle α. Assume their polarizations to be linear, one laying in the plane of the wavevectors, the other one orthogonal to it [16]. Then the Poynting vector exhibits in general a quite perplexing component normal to the plane of the wavevectors!

the tragicomic embarrass of the theoretical physicist in the exhilarating tale written by J.L. Synge [17] and aptly entitled "On the present status of the electromagnetic energy-tensor".[5)]

Another argument for selecting the energy tensor relies on a *reductio ad vacuum* of a different sort, namely to the vacuum of special relativity, supposed to prevail at a microscopic scale. This is one of the pillars of a far-reaching program, whose initiator was Lorentz himself [18], and which has been endorsed by many adherents [19]-[23]. In particular the issue of the energy tensor has been attacked by De Groot and Suttorp in a remarkable series of papers [24] aiming at a clean derivation, explicitly conforming to the tenets of special relativity. But they too do not seem to escape a simple criticism, that one *e.g.* reads off an earlier paper by Ott [25].

Imagine that one wishes to produce the macroscopic electromagnetic energy tensor by starting from the vacuum tensor that supposedly holds at a microscopic scale, in keeping with Lorentz' idea. One is then confronted with the task of calculating some sort of macroscopic average, like

$$< T_{ik} >=< F_i{}^n F_{kn} - \tfrac{1}{4}\eta_{ik}F^{mn}F_{mn} >, \tag{38}$$

where the brackets are used to indicate that some process of macroscopic averaging has been performed. Let us assume that the microscopically fluctuating electromagnetic field F_{ik} can be split into a microscopically smooth average term $\bar{F}_{ik} \equiv < F_{ik} >$, and a term ξ_{ik}, that oscillates at a microscopic scale:

$$F_{ik} = \bar{F}_{ik} + \xi_{ik}. \tag{39}$$

Then, obviously:

$$< T_{ik} >= \bar{F}_i{}^n \bar{F}_{kn} - \tfrac{1}{4}\eta_{ik}\bar{F}^{mn}\bar{F}_{mn} + < \xi_i{}^n \xi_{kn} - \tfrac{1}{4}\eta_{ik}\xi^{mn}\xi_{mn} > . \tag{40}$$

The first two terms at the right-hand side form the energy tensor of the macroscopic field $\bar{F}_{ik}$, whose constitutive relations are just the ones that hold for the macroscopic vacuum. The evaluation of the last term looks instead like a desperate task, since it would require a detailed knowledge of correlations between microscopic fields for which we have nothing better than hypothetical microscopic models. But suppose anyway that this task can be accomplished: we still have to split $< T_{ik} >$ into a part pertaining to the medium, and a part pertaining to the macroscopic field that with that medium is interacting. No wonder, then, if the most ingenious efforts in this direction, spread on a time span that covers several decades, do not seem to converge towards a unique answer, as one gathers from the study of the literature mentioned above.

7. Conclusion.

Given the bleak status in which the issue of the energy tensor for the electromagnetic field in matter finds itself,[6)] a little paradigm shift suggests itself: rescuing from oblivion the admittedly macroscopic, but clear-cut general relativistic argument given by Gordon, that uniquely selects Abraham's tensor as the electromagnetic energy tensor for a material medium which is homogeneous and isotropic in its rest frame.

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5) In that tale a theoretical physicist is bluntly confronted with the questions posed by simple Simon, a man who farms chicken and wants to understand for good why in the incubator built by himself with static, mutually orthogonal electric and magnetic fields his poor chicken do not appear to benefit from the theoretically predicted electromagnetic energy flow.

6) Not substantially bettered by the improved description of microscopic matter that can be achieved through quantum mechanics.

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