Extracavity quantum vacuum radiation from a single qubit

S. De Liberato,[1,2] D. Gerace,[3] I. Carusotto,[4] and C. Ciuti[1,*]

[1]*Laboratoire Matériaux et Phénomènes Quantiques, Université Paris Diderot-Paris 7 and CNRS, UMR 7162,*
75205 Paris Cedex 13, France

[2]*Hypios Research Department, 187 rue du Temple, 75003 Paris, France*

[3]*CNISM and Dipartimento di Fisica "A. Volta," Università di Pavia, I-27100 Pavia, Italy*

[4]*BEC-CNR-INFM and Dipartimento di Fisica, Università di Trento, I-38050 Povo, Italy*

We present a theory of the quantum vacuum radiation that is generated by a fast modulation of the vacuum Rabi frequency of a single two-level system strongly coupled to a single cavity mode. The dissipative dynamics of the Jaynes-Cummings model in the presence of antirotating-wave terms is described by a generalized master equation including non-Markovian terms. Peculiar spectral properties and significant extracavity quantum vacuum radiation output are predicted for state-of-the-art circuit cavity quantum electrodynamics systems with superconducting qubits.

Cavity quantum electrodynamics (CQED) is a very exciting and active research field of fundamental quantum physics, characterized by an unprecedented control of light-matter interaction down to the single-quantum level [1]. A number of different systems and a wide range of electromagnetic frequencies are presently being explored in this context, including Rydberg atoms in superconductor microwave cavities [1], alkali metal atoms in high-finesse optical cavities [2,3], single-quantum dots in semiconductor optical nanocavities [4,5], and superconductor Cooper pair quantum boxes in microwave strip-line resonators [6–9].

Most of the research in CQED has so far concerned systems whose properties are slowly varying in time with respect to the inverse resonance frequency of the cavity mode. Only very recently, experiments with semiconductor microcavities [10] have demonstrated the possibility of modulating the vacuum Rabi coupling on a time scale comparable to a single oscillation cycle of the field. For this novel regime, theoretical studies have anticipated the possibility of observing a sizeable emission of quantum vacuum radiation [11] via a process that is closely reminiscent of the still elusive dynamical Casimir effect [12]: the modulation of the Rabi coupling provides a modulation of the effective optical length of the cavity, and it is analogous to a rapid displacement of the cavity mirrors.

A recent paper [13] has applied this general scheme to a Jaynes-Cummings (JC) model in the presence of a fast modulation of the artificial atom resonance frequency. However, as the theoretical model did not include dissipation, the predictions were limited to short times and were not able to realistically describe the system steady state. In particular, no quantitative estimation of the extracavity radiation intensity was provided.

In the present article, we introduce a full quantum theory to describe the nonadiabatic response of a JC model including both the antirotating-wave terms of the light-matter interaction and a realistic dissipative coupling to the environ-

ment. While the former terms are responsible for the generation of photons out of the nontrivial ground state [11,14], radiative coupling to the external world is essential to detect the generated photons as emitted radiation. Our attention will be focused on the most significant case of harmonic temporal modulation of the vacuum Rabi coupling of superconducting qubits in circuit CQED systems: for fast, yet realistic [15,16] modulations of the vacuum Rabi coupling, the photon emission turns out to be significant even in the presence of a strong dissipation. Furthermore, in contrast to other systems that have been proposed in view of observing the dynamical Casimir effect [17–20], the intrinsic quantum nonlinear properties of the two-level system should allow experimentalists to isolate the vacuum radiation from the parametric amplification of pre-existing thermal photons.

In addition to its importance concerning the observation of the dynamical Casimir effect, the theory developed here appears of great interest also from the general point of view of the quantum theory of open systems [21]. As a consequence of the antirotating-wave terms in the Hamiltonian, even the ground state of the system contains a finite number of photons. In order for the theory not to predict unphysical radiation from these bound virtual photons [22], the theoretical model has to explicitly take into account the colored nature of the dissipation bath. This suggests circuit CQED systems as unique candidates for the study of non-Markovian effects in the dissipative dynamics of open quantum systems.

A sketch of the system under consideration is shown in the left panel of Fig. 1. The theoretical description of its dynamics is based on the Jaynes-Cummings Hamiltonian

$$\hat{H}(t) = \hbar\omega_0\hat{a}^\dagger\hat{a} + \hbar\omega_{ge}\hat{c}^\dagger\hat{c} + \hbar g(t)(\hat{a} + \hat{a}^\dagger)(\hat{c} + \hat{c}^\dagger), \quad (1)$$

whose ladder of eigenstates is schematically drawn in the right panel. Here, $\hat{a}^\dagger$ is the bosonic creation operator of a cavity photon and $\hat{c}^\dagger$ is the raising operator describing the excitation of the two-level system (qubit), $\hat{c}^\dagger|g\rangle = |e\rangle$, where $|g\rangle$ and $|e\rangle$ are its ground and excited states, respectively; ω_0 is the bare frequency of the cavity mode and ω_{ge} is the qubit transition frequency. The term proportional to $g(t)$ describes
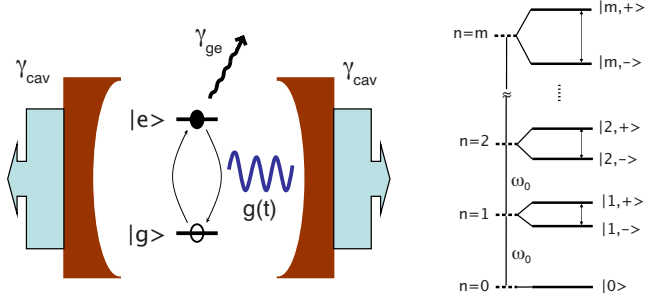
*cristiano.ciuti@univ-paris-diderot.fr

FIG. 1. (Color online) Left panel: sketch of the system under consideration. A single two-level system (qubit) is strongly coupled to a single cavity mode. A possible realization of such a device consists of a Cooper pair quantum box embedded in a microwave resonator. Right panel: schematic representation of the JC ladder of eigenstates of the isolated system in the absence of modulation, dissipation and anti-rotating-wave terms. In this limit, the eigenstates $|n, \pm\rangle = (|n, g\rangle \pm |n-1, e\rangle)/\sqrt{2}$ have energies $E_{n,\pm} = n\omega_0 \pm \sqrt{n}g_0$.

the vacuum Rabi coupling between the two-level system and the cavity mode and fully includes those antiresonant non-rotating-wave processes that are generally neglected in the so-called rotating-wave approximation (RWA). Namely, the antiresonant (non-rotating-wave) part of the interaction Hamiltonian is $\hbar g(t)(\hat{a}\hat{c} + \hat{a}^{\dagger}\hat{c}^{\dagger})$.

While the RWA has provided an accurate description of most physical CQED systems [1–9], it becomes inaccurate as soon as one enters the so-called ultrastrong coupling regime, i.e., when the Rabi coupling, g, is comparable to the resonance frequencies, ω_0 and ω_{ge}. This regime has been recently achieved in a solid-state device consisting of a dense two-dimensional electron gas with an intersubband transition coupled to a microcavity photon mode [23]. Values of the g/ω_{ge} ratio of the order of 0.01 (approaching the so-called fine-structure constant limit) have been recently observed also in circuit CQED systems, and even larger values have been predicted for more recent unconventional coupling configurations [24]. While the non-rotating-wave terms only provide a quantitative correction in the static case with a constant g, their effect is essential in the experimentally novel [10,15] regime, where the Rabi frequency $g(t)$ is modulated in time at frequencies comparable or higher than the qubit transition frequency. In fact, a nonadiabatic modulation of the non-rotating-wave terms can in this case lead to the emission of quantum vacuum radiation, a phenomenon which would be completely absent if these terms were neglected. In the absence of antiresonant terms, the light-matter Hamiltonian interaction does not couple states with different excitation number n (light+matter) even in presence of temporal modulation. In particular, if the antiresonant terms are not included in the Hamiltonian, no photons can excited if the initial state is the vacuum. As we show in this article, a significant amount of quantum vacuum radiation with peculiar spectral features can be already expected for moderate values of g/ω_{ge}, i.e., compatible with already existing circuit CQED samples.

In order to fully describe the quantum dynamics of the system, the JC model has to be coupled to its environment. A simple description involves two thermal baths, corresponding to the radiative and nonradiative dissipation channels. The non-Markovian nature of the baths is taken into account by means of a so-called second-order time-convolutionless projection operator method [21], which gives a master equation of the general form,

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[\hat{H}, \rho] + \sum_{j=\text{cav,ge}} (\hat{U}_j \rho \hat{S}_j + \hat{S}_j \rho \hat{U}_j^{\dagger} - \hat{S}_j \hat{U}_j \rho - \rho \hat{U}_j^{\dagger} \hat{S}_j),$$

(2)

where $\hat{S}_{\text{cav}} = (\hat{a} + \hat{a}^{\dagger})/\hbar$, $\hat{S}_{\text{ge}} = (\hat{c} + \hat{c}^{\dagger})/\hbar$, and $\hat{U}_j$ are given by integral operators as

$$\hat{U}_j(t) = \int_0^{\infty} v_j(\tau) e^{-i\hat{H}(t)\tau} \hat{S}_j e^{i\hat{H}(t)\tau} d\tau,$$

(3)

$$v_j(\tau) = \int_{-\infty}^{\infty} \frac{\gamma_j(\omega)}{2\pi} \{n_j(\omega)e^{i\omega\tau} + [n_j(\omega) + 1]e^{-i\omega\tau}\}d\omega.$$

(4)

The energy-dependent loss rates, $\gamma_j(\omega)$, for the cavity (i.e., $j=\text{cav}$) and for the qubit transition ($j=\text{ge}$) are related to the density of states at energy $\hbar\omega$ in the baths, and thus they must be set to zero for $\omega<0$. In the numerical simulations, we used the simple form $\gamma_j(\omega) = \gamma_j\Theta(\omega)$ for the nonwhite loss rates, where $\Theta(\omega)$ is the Heaviside step function. In the following, the background number of thermal excitations at energy $\hbar\omega$ in the corresponding bath will be set as $n_j(\omega)=0$. The *usually* employed master equation (see, e.g., Ref. [25]) is recovered from Eq. (2) if one assumes the baths to be perfectly white, i.e., $\gamma_j(\omega) = \gamma_j$. However, by doing so, one implicitly introduces unphysical negative-energy radiative photon modes, which incorrectly lead to the unphysical emission of light out of the vacuum state even in absence of any modulation [25].

In the present article, we shall focus on the steady state of the system under a harmonic modulation of the form

$$g(t) = g_0 + \Delta g \sin(\omega_{\text{mod}}t),$$

(5)

where Δg and ω_{mod} are the modulation amplitude and frequency, respectively. Direct application to the present JC model of the input-output formalism, e.g., discussed in Ref. [22], leads to the following expression for the spectral density of extracavity photons emitted per unit time:

$$\mathcal{S}(\omega) = \frac{\gamma_{\text{cav}}(\omega)}{2\pi} G(\omega)$$

(6)

in terms of the intracavity field spectrum, $G(\omega)$. As a consequence of the (harmonic) modulation $g(t)$, the spectrum $G(\omega)$ involves a temporal average over the modulation period $T_{\text{mod}} = 2\pi/\omega_{\text{mod}}$,

$$G(\omega) = \frac{1}{T_{\text{mod}}} \int_0^{T_{\text{mod}}} dt \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \text{Tr}\{\hat{a}^{\dagger}(t+\tau)\hat{a}(t)\rho\}.$$

(7)

The cavity field operators $\hat{a}(t)$ are defined here in the Heisenberg picture. The total number of extracavity photons emitted per unit time is given by the spectral integral $R_{\text{em}} = \int_{-\infty}^{\infty} d\omega \mathcal{S}(\omega)$. This formula is to be contrasted with the
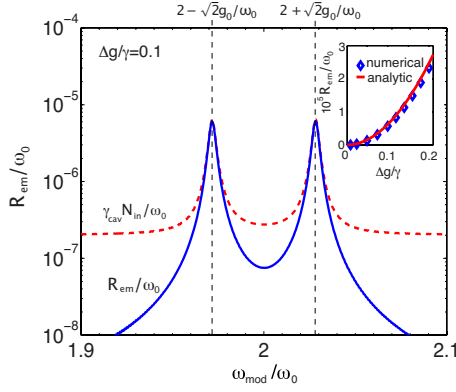
FIG. 2. (Color online) Extracavity photon emission rate R_{em} (units of ω_0) as a function of the modulation frequency, ω_{mod}, for a modulation amplitude of the vacuum Rabi frequency $\Delta g/\gamma=0.1$. Parameters: $\omega_0=\omega_{ge}$; $\gamma=\gamma_{cav}=\gamma_{ge}=0.002\omega_0$; $g_0=0.02\omega_0$. For comparison, the dashed line shows the extracavity emission rate $\gamma_{cav}N_{in}$ (where N_{in} is the steady-state intracavity photon number) that would be predicted by the Markovian approximation: note the unphysical prediction of a finite value of the emission even far from resonance. The inset shows the dependence of the photon emission rate on the modulation amplitude, calculated both numerically and analytically (see later in the text), for $\omega_{mod}=\omega_{2,-}$.

one giving the time average of the intracavity photon number, $N_{in}=T_{mod}^{-1}\int_0^{T_{mod}}dt\,\mathrm{Tr}\{\hat{a}^\dagger(t)\hat{a}(t)\rho\}$.

The master Eq. (2) is numerically solved by representing $\hat{a}$ and $\hat{c}$ on a basis of Fock number states. The operators $\hat{S}$ and $\hat{U}$ are also numerically built and all the time evolutions are performed by a Runge-Kutta algorithm. Examples of numerical results are shown in Figs. 2 and 3 for the resonant case ($\omega_0=\omega_{ge}$), but we have checked that the qualitative features do not change when we introduce a finite detuning. Realistic parameters for circuit CQED systems are considered, as indicated in the caption.

In Fig. 2 we show the steady-state rate of emitted photons as a function of the modulation frequency, ω_{mod}, for the case of a weak modulation amplitude, $\Delta g/\gamma\ll 1$. In this regime, the spectra are dominated by two resonant peaks close to $\omega_{mod}\simeq\omega_{2,\pm}=2\omega_0\pm\sqrt{2}g_0$. Thanks to the relatively small value of $g_0/\omega_0=0.02$ considered here, the position of the two peaks can be interpreted within the standard RWA in terms of transitions from the vacuum state to the doubly-excited states of the JC ladder in the isolated system, $|2,\pm\rangle$; the non-rotating-wave terms in the Hamiltonian that are responsible and essential for the quantum vacuum radiation instead provide only a minor correction to the spectral position of the peaks [26]. Stronger modulations lead to quadratically larger emission intensities, then (not shown) to the appearance of additional weaker peaks around $\omega_{mod}\simeq 2\omega_0\pm\sqrt{2/n}g_0$ as a consequence of higher ($n\geq 2$) order processes, and finally (not shown) to a distortion of the whole spectrum as a result of significant spectral shifts and mixing of the dressed states.

As typical of the dynamical Casimir effect, the periodic modulation of the system parameters is only able to create pairs of excitations out of the vacuum state. However, in contrast to the usual case of (almost) noninteracting photons
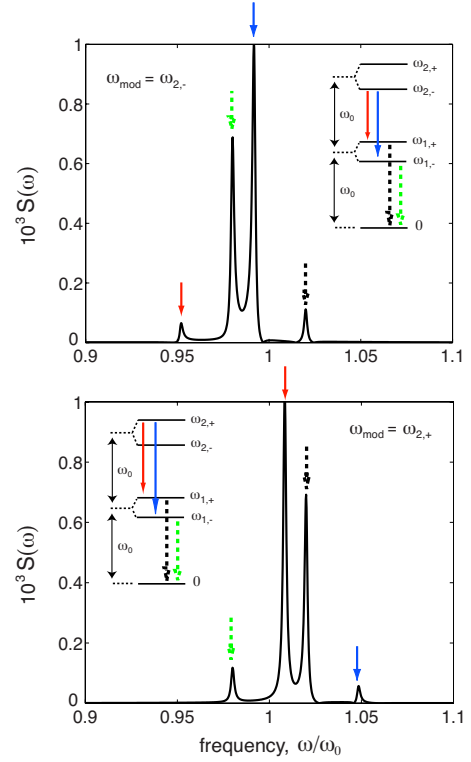

FIG. 3. (Color online) Spectral density $\mathcal{S}(\omega)$ of extracavity photons emitted per unit time for given values of the modulation frequency: $\omega_{mod}=\omega_{2,-}$ (top panel), $\omega_{mod}=\omega_{2,+}$ (bottom panel). The modulation amplitude is $\Delta g/\gamma=0.1$. The insets illustrate the optical transitions responsible for the different emission lines that are visible in the main panels. For the sake of clarity, the level spacings in the insets are not in scale.

or bosonic polaritons [17–19], the nonlinear saturation of the two-level system is crucial here to determine the position of the peaks. This remarkable fact provides a unique spectral signature to separate the vacuum radiation from spurious processes such as the parametric amplification of thermal radiation. In fact, due to the anharmonicity of the Jaynes-Cummings spectrum, the resonant modulation frequency for the process having the ground state (vacuum) as initial state is different from other processes having a (thermal) excited state as initial state.

The conceptual difference between the emission rate R_{em} and in-cavity photon number N_{in} is illustrated in Fig. 2: standard Markovian theories would in fact predict the emission rate to be rigorously proportional to the intracavity photon number. Even though a reasonable agreement is observed around the peaks, this approximation leads to the unphysical prediction of a finite emission even in the absence of a modulation ($\Delta g=0$) or for a modulation very far from resonance. Inclusion of the non-Markovian nature of the baths is able to eliminate this pathology by correctly distinguishing the virtual bound photons that exist even in the ground state from the actual radiation [11,22].

The emission spectra $\mathcal{S}(\omega)$ at a fixed and resonant value of the modulation frequencies $\omega_{mod}=\omega_{2,\pm}$ are shown in the two panels of Fig. 3. Thanks to the relatively weak value $\Delta g/g_0=0.01$ ($\Delta g/\gamma=0.1$) of the modulation amplitude con-

sidered here, the position of the main emission lines can be again understood in terms of transitions between eigenstates of the JC ladder. As shown in the insets, two spectral lines (full lines in the schemes) correspond to radiative decay (emission) of the $|2, \pm\rangle$ states into the lower $|1, \pm\rangle$ states of the JC ladder, while the other two emission peaks (dashed lines) correspond to the radiative decay of the $|1, \pm\rangle$ states into the ground state. This interpretation is confirmed by the observation that the position of the former (latter) lines depends (does not depend) on the specific value of the modulation frequency ω_{mod} chosen. The significant difference of spectral weight between the lines is due to interference effects in the radiative matrix element between JC eigenstates, $\langle 1, \pm | \hat{a} | 2, \pm \rangle$.

In the weak modulation intensity regime, the behavior of the exact numerical results can be understood in terms of a simplified two-state model. When the modulation frequency is close to resonance with one of the $\omega_{mod} = \omega_{2,\pm}$ peaks, the dynamics of the system is mostly limited to the $|0\rangle$ and $|2, \pm\rangle$ states, all other states in the JC ladder being far off-resonant [3]. The non-rotating-wave terms proportional to the modulation of the vacuum Rabi coupling in Eq. (5) are responsible for an effective coupling between such two states, quantified by $\Omega_R \simeq \Delta g / \sqrt{2}$ [19]. As a result, the probability of being in the excited state has the usual Lorentzian line shape

$$P_{|2,\pm\rangle} \simeq \frac{(\Delta g)^2 / 2}{\Gamma^2 + 4\delta_{2,\pm}^2}. \tag{8}$$

Here, $\Gamma = [\gamma_{ge} + 3\gamma_{cav}]/2$ is the total (radiative +nonradiative) decay rate of the excited $|2, \pm\rangle$ state (in the case $\omega_{cav} = \omega_{ge}$), and $\delta_{2,\pm} = \omega_{mod} - \omega_{2,\pm}$ is the detuning of the modulation frequency. By considering all the possible emission cascades (see insets of Fig. 3), the radiative emission rate in the neighborhood of a peak is then approximately given by

$$R_{em} \simeq P_{|2,\pm\rangle} \gamma_{cav} \frac{3\gamma_{cav} + 2\gamma_{ge}}{\gamma_{cav} + \gamma_{ge}}. \tag{9}$$

This analytical expression is in excellent agreement with the exact numerical results for $\omega_{mod} \simeq \omega_{2,\pm}$, as shown in the inset of Fig. 2. It is interesting to note that for typical parameters taken from state-of-the art circuit CQED devices, such as a resonance frequency $\nu_0 = \omega_0 / 2\pi \sim 7$ GHz [9] and (overestimated) decay rates $\gamma / 2\pi = 14$ MHz, a resonant, yet quite small modulation amplitude $\Delta g / \gamma = 0.1$ can already lead to a sizeable emission intensity, $R_{em} \simeq 4 \times 10^4$ photons/second. As clearly shown by the analytical expression in Eq. (9), a further enhancement of the emission rate can be obtained for much smaller decay rates, such as $\gamma / 2\pi \lesssim 1$ MHz recently measured in the latest experiments [7–9].

In conclusion, we have presented and solved a complete theory of the quantum vacuum emission that is generated from a single mode cavity with an embedded two-level system when the vacuum Rabi frequency of the light-matter interaction is modulated at frequencies comparable to the cavity (emitter) resonance frequency. Our theory fully takes into account the non-rotating-wave terms of the light-matter interaction that are responsible for the quantum vacuum emission, as well as the radiative and nonradiative dissipation channels. This has required extending the standard master-equation treatment to include non-Markovian effects due to the necessarily colored nature of any realistic dissipation bath. The sizable value of the emission intensity that results from our theoretical predictions suggests the promise of superconductor Cooper quantum boxes in microwave resonators for studies of quantum vacuum radiation phenomena.

Note added in proof. Recently, another, similar investigation on quantum vacuum radiation and the dynamical Casimir effect has been published [27].

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