Spontaneous four-wave mixing in microring resonators

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We consider spontaneous four-wave mixing in a microring resonator, presenting photon-pair generation rates and biphoton wave functions. We show how generation rates can be simply predicted from the performance of the device in the classical regime and that a wide variety of biphoton wave functions can be achieved by varying the pump pulse duration.

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Traditional photon-pair sources require bulk optical elements that limit scalability beyond a research laboratory. More compact structures, often initially used to enhance a stimulated (classical) nonlinear optical process, have recently attracted interest as potential integrated or "on-chip" sources of quantum correlated photon pairs [1–9]. These include structured media such as microtorroids, photonic crystal cavities, and microring resonators. Yet, while microring resonators have been studied theoretically with respect to efficiencies for second-harmonic generation [10], spontaneous parametric downconversion [11], and classical four-wave mixing (FWM) [12], spontaneous FWM in a microring resonator has not been thoroughly investigated. Previous theoretical studies of spontaneous FWM considered fiber geometries [13,14], and only recently has photon-pair generation from a silicon microring resonator been demonstrated [4]. However, as advances in fabrication technology continue [15,16], lower losses and greater compatibility with existing infrastructure will lead to tailored, efficient, and integrated sources of photon pairs. Particularly important is understanding the range of quantum correlated states that can be generated.

Here we consider spontaneous and stimulated FWM in a microring resonator side coupled to a single-channel waveguide [see Fig. 1(a)]. We study quantum and classical processes within the same framework [17] and relate the efficiency of photon-pair generation via spontaneous FWM to that of the corresponding stimulated (classical) process. We calculate the quantum state at the output of the structure for a given input state, including the possibility of the generation of multiple pairs, limited only by the undepleted pump approximation; for photons generated at a single pair of resonances, we show that it is easy to control the Schmidt number [18] by varying the temporal duration of the pump pulse.

As usual, we treat the coupling between the ring and the channel as weak and occurring at a single point [11]. The FWM processes are described by the third-order nonlinear Hamiltonian $\hat{H} = \hat{H}^{\rm ch} + \hat{H}^{\rm cp} + \hat{H}^{R}$, where

$$\hat{H}^{\rm ch} = \sum_{\mu} \left\{ \begin{array}{c} \hbar\omega_\mu \int {\rm d}z \hat{\psi}_\mu^\dagger(z)\hat{\psi}_\mu(z) \\ +\frac{i}{2}\hbar v_\mu \int {\rm d}z \left(\frac{\partial \hat{\psi}_\mu^\dagger(z)}{\partial z}\hat{\psi}_\mu(z) - h.c.\right) \end{array} \right\},$$

$$\hat{H}^{\rm cp} = \sqrt{2\pi}\hbar \sum_\mu \{c_\mu \hat{b}_\mu^\dagger \hat{\psi}_\mu(0) + h.c.\},$$

$$\hat{H}^{R} = \sum_\mu \hbar\omega_\mu \hat{b}_\mu^\dagger \hat{b}_\mu - \sum_{\mu_1,\mu_2,\mu_3,\mu_4} S_{\mu_1\mu_2\mu_3\mu_4} \hat{b}_{\mu_1}^\dagger \hat{b}_{\mu_2}^\dagger \hat{b}_{\mu_3} \hat{b}_{\mu_4}, \qquad (1)$$

with $\hat{\psi}_\mu$ being the channel waveguide field operator, $\hat{b}_\mu$ the ring-resonator operator, ω_μ the eigenfrequency, and v_μ the group velocity of mode $\mu = (m, N)$, where m identifies the transverse mode and N its associated ring-resonance order; c_μ is a coupling constant defined later, $S_{\mu_1\mu_2\mu_3\mu_4}$ describes the nonlinear effects in the resonator and is given by a straightforward extension of an earlier expression [17], and we have assumed that the free spectral range of the ring resonances is large enough that the $\hat{\psi}_\mu$ associated with one resonance commutes with all the others. Then, following [10,11], we consider a coherent state $|\psi_{\rm in}\rangle = e^{\alpha \hat{A}_{\mu_P}^\dagger - h.c.}|{\rm vac}\rangle$ in the channel at the point of coupling, where $\hat{A}_{\mu_P}^\dagger = \int {\rm d}k \phi_P(k)\hat{a}_{\mu_P}^\dagger(k)$ and $\hat{a}_{\mu_P}(k) = (2\pi)^{-\frac{1}{2}} \int \hat{\psi}_{\mu_P}(z) e^{-ikz}$, such that $|\alpha|^2$ gives the number of photons in the pulse. We work in the undepleted pump approximation, assuming that the mode profiles and group velocities of the modes involved vary little over the frequency range of interest and find the state of generated photons resulting from a spontaneous FWM process. In the present calculation, we neglect loss mechanisms, such as absorption, scattering, and Raman effects, as well as self- and cross-phase modulation

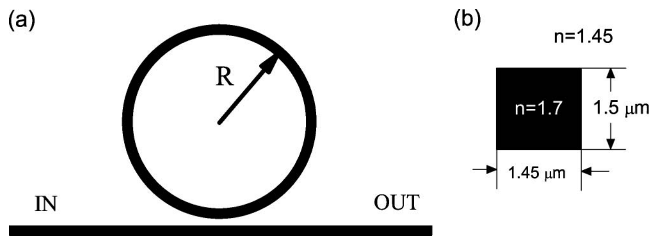

Fig. 1. (a) Schematic of a single-channel ring resonator. (b) Cross section of the channel.

effects. We plan to include them in future work, but they can be safely ignored in the structure we consider [7].

Regardless of how many photons are generated, within our approximations the state of generated photons resulting from $|\psi_{\rm in}\rangle$ is a two-mode squeezed vacuum. We write it as $|\psi_{\rm gen}\rangle = e^{\beta \hat{C}_{II}^{\dagger} - h.c.}|{\rm vac}\rangle$, where $\hat{C}_{II}^{\dagger}|{\rm vac}\rangle = \frac{1}{\sqrt{2}}\sum_{\mu_1,\mu_2}\int {\rm d}k_1{\rm d}k_2\phi_{\mu_1\mu_2}(k_1,k_2)\hat{a}_{\mu_1}^{\dagger}(k_1)\hat{a}_{\mu_2}^{\dagger}(k_2)|{\rm vac}\rangle$ is a normalized two-photon state characterized by the biphoton wave function $\phi_{\mu_1\mu_2}(k_1,k_2)$. In deriving $|\psi_{\rm gen}\rangle$, the main subtlety (cf. [10]) is that the exact expression for the state of the output field contains noncommuting operators and cannot simply be split into an undepleted pump field and a generated field; we use the Baker–Campbell–Hausdorff formula [19] to keep only leading-order terms in the exponential that involve two-photon creation operators. We then define $|\beta_{\mu_1\mu_2}|^2 = |\beta|^2\int {\rm d}k_1{\rm d}k_2|\phi_{\mu_1\mu_2}(k_1,k_2)|^2$, and thus $|\beta|^2 = \sum_{\mu_1,\mu_2}|\beta_{\mu_1\mu_2}|^2$. The biphoton wave function is symmetric, $\phi_{\mu_1\mu_2}(k_1,k_2) = \phi_{\mu_2\mu_1}(k_2,k_1)$, so for $|\beta_{\mu_1\mu_2}| \ll 1$, the average number of photon pairs generated in modes μ_1, μ_2 per pump pulse is $2|\beta_{\mu_1\mu_2}|^2$.

Here we consider only Type I FWM, in which the pump, signal, and idler are all TM polarized; we assume a Gaussian pump waveform sufficiently narrow to excite only the resonance order N_P:

$$\phi_P(k) = \exp(-(k-k_{N_P})^2/(4\Delta_k^2))/(\sqrt{2\pi}\Delta_k)^{1/2},$$

where Δ_k is the wavenumber associated with that resonance order, and $\Delta_k = \sqrt{2\ln(2)}/v_{N_P}T$, where T is the intensity FWHM of the pulse in time. This implies that if one photon is generated near the k associated with resonance order N on one side of the pump, the other will be generated near the k associated with resonance order $\bar{N} = 2N_P - N$ on the other side, thus reducing double sums over the modes to single sums. Our approach can deal with more complicated situations, including nonclassically described pump pulses, but the above assumptions greatly simplify the notation; without ambiguity we now write N in place of μ.

With these considerations, we calculate the relation between $2|\beta_{N\bar{N}}|^2$ and $|\alpha|^2$ to be

$$2|\beta_{N\bar{N}}|^2 = 4\gamma^2\hbar^2\omega_N\omega_{\bar{N}}|\alpha|^4\frac{\Omega_{N_P}^2\Omega_N\Omega_{\bar{N}}v_{N_P}^4 v_N v_{\bar{N}}}{L^2\pi^2}J_{N\bar{N}}, \quad (2)$$

where

$$J_{N\bar{N}} = \int {\rm d}k_1{\rm d}k_2 \left|\int {\rm d}k \frac{\phi_P(k)\phi_P(\Delta\omega(k_1,k_2)/v_{N_P}-k)(\Delta\Omega_{N_P}(k))^{-1}}{\Delta\Omega_N(k_1)\Delta\Omega_{\bar{N}}(k_2)\Delta\Omega_{N_P}(\Delta\omega(k_1,k_2)/v_{N_P}-k)}\right|^2,$$

$\Delta\omega(k_1,k_2) = v_N k_1 + v_{\bar{N}}k_2 + \omega_N + \omega_{\bar{N}} - 2\omega_{N_P}$, and $\Delta\Omega_N(k) = iv_N(k-k_N) - \Omega_N$. Here $L = 2\pi R$ is the ring circumference, with half the FWHM of a resonance linewidth in (circular) frequency $\Omega_N = \pi|c_N|^2/v_N$, simply related to the usual self-coupling constant, σ_N, via $\Omega_N = (1-\sigma_N)v_N/L$; $\gamma = 3\bar{\chi}_3\omega_{N_P}/(4\varepsilon_0 v_{N_P}^2\bar{n}^4\mathcal{A}_{\rm eff})$ is the usual nonlinear parameter; $\mathcal{A}_{\rm eff}$ is an effective area [10]; $\bar{n}$ and $\bar{\chi}_3$ are, respectively, a refractive index and third-order nonlinearity introduced solely for convenience [17].

We consider long (short) pulses, for which $\Omega_{N_P}T \gg 1$ ($\Omega_{N_P}T \ll 1$). For $\Omega_{N_P}T \gg 1$, we find a normalized biphoton probability density

$$|\beta\phi_{N\bar{N}}(k_1,k_2)/\beta_{N\bar{N}}|^2 = \frac{e^{-(\Delta\omega(k_1,k_2)/v_{N_P}-2k_{N_P})^2/(4\Delta_k^2)}}{J_{N\bar{N}}\Omega_{N_P}^4\left|\Delta\Omega_N(k_1)\right|^2\left|\Delta\Omega_{\bar{N}}(k_2)\right|^2}, \quad (3)$$

with

$$J_{N\bar{N}} = \frac{2\Delta_k\pi^{3/2}(\Omega_N+\Omega_{\bar{N}})v_{N_P}(v_N v_{\bar{N}}\Omega_N\Omega_{\bar{N}}\Omega_{N_P}^4)^{-1}}{(\Delta\omega(k_N,k_{\bar{N}})-2v_{N_P}k_{N_P})^2+(\Omega_N+\Omega_{\bar{N}})^2},$$

whereas for $\Omega_{N_P}T \ll 1$

$$|\beta\phi_{N\bar{N}}(k_1,k_2)/\beta_{N\bar{N}}|^2 = \frac{2\pi(v_{N_P}^2(\Delta\omega(k_1,k_2)/v_{N_P}-2k_{N_P})^2+4\Omega_{N_P}^2)^{-1}}{J_{N\bar{N}}\Delta_k^2 v_{N_P}^2\left|\Delta\Omega_N(k_1)\right|^2\left|\Delta\Omega_{\bar{N}}(k_2)\right|^2}, \quad (4)$$

with

$$J_{N\bar{N}} = \frac{\pi^3(\Omega_N+\Omega_{\bar{N}}+2\Omega_{N_P})(\Delta_k^2 v_N v_{\bar{N}}v_{N_P}^2\Omega_N\Omega_{\bar{N}}\Omega_{N_P})^{-1}}{(\Delta\omega(k_N,k_{\bar{N}})-2v_{N_P}k_{N_P})^2+(\Omega_N+\Omega_{\bar{N}}+2\Omega_{N_P})^2}.$$

We now apply these results to a specific 47.5 μm radius structure (see Fig. 1) [7]. We calculate that, for N (and thus $\bar{N}$) not too far from $N_P = 310$, we have $v_{N_P} \approx v_N \approx v_{\bar{N}} \equiv v = \frac{c}{\bar{n}} = 171$ μm/ps, and $\Omega_{N_P} \approx \Omega_N \approx \Omega_{\bar{N}} \equiv \Omega = 2\pi(0.74\ {\rm GHz})$; we also Taylor expand the dispersion relation for the ring according to $\omega_N = \omega_{N_P} + v_{N_P}(k_N-k_{N_P}) + \Xi(k_N-k_{N_P})^2$, where $\Xi = 0.01$ μm^2/ps. In Fig. 2, we plot the biphoton probability densities corresponding to each of the above limits, with $T = 5$ ns and $T = 5$ ps, for $N = 311$. The long-pulse-limit biphoton probability density is long and narrow, whereas the short-pulse limit is rather circular. Quantifying this, the Schmidt number for the biphoton wave function, [18] $K \equiv (\sum_i \lambda_i^2)^{-1}$, where the λ_i are the eigenvalues of the reduced density operator, is 10.00 for the 5 ns pulse, naturally becomes greater for longer pulses, and is 1.09 for the 5 ps pulse.

In a classical undepleted pump (and signal) calculation starting from the same microring resonator Hamiltonian, the idler power generated for a given cw input pump (signal) power near resonance N_P (N) is

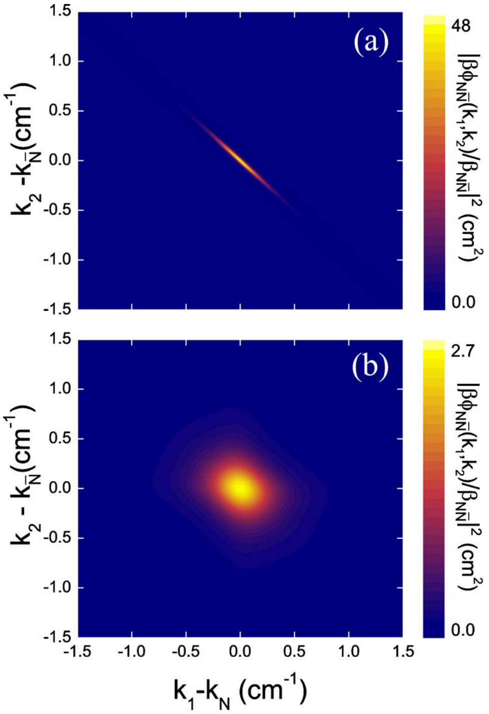

Fig. 2. (Color online) Biphoton probability density for a pump pulse centered at $N_P = 310$ with (a) $T = 5$ ns and (b) $T = 5$ ps. We consider $N = 311$ and $\bar{N} = 309$.

$$P_{\bar{N}} = (\gamma P_{N_P} L)^2 |F_N|^2 |F_{\bar{N}}|^2 |F_{N_P}|^4 P_N, \qquad (5)$$

where the field-enhancement factor is $F_N = i(1 - \sigma_N^2)^{1/2}/(1 - \sigma_N e^{iL\bar{n}\delta_N/c})$ for frequency detuning δ_N from resonance N. We note that this expression agrees with previous results [7,12], in the limit of no loss and weak coupling, $\sigma_N \approx 1$. If we had instead considered the true cw limit in our quantum calculation, we would have found, using the same assumptions as above,

$$2|\beta_{N\bar{N}}|^2 = \left(\frac{\gamma P_{N_P}}{L}\right)^2 \frac{16\omega_N \omega_{\bar{N}} v^3 \Lambda}{\omega_{N_P}^2 \Omega((\omega_N + \omega_{\bar{N}} - 2\omega_{N_P})^2 + 4\Omega^2)}.$$

Here we have taken the pump waveform to be a top-hat function, of length Λ in real space in the long pulse limit, $\Omega_{N_P} \frac{\Lambda}{v} \gg 1$, and identified $P_{N_P} = \hbar\omega_{N_P} v|\alpha|^2/\Lambda$ as the average pump power. Strictly speaking, this corresponds to the average number of generated photon pairs generated near resonance orders N and $\bar{N}$ per unit time, but multiplying by $\hbar\omega_{\bar{N}} v/\Lambda$ will give the average power of all photons generated near the resonance order $\bar{N}$:

$$P_{\bar{N}} = \left(\frac{\gamma P_{N_P}}{L}\right)^2 \frac{16\hbar\omega_N^2 \omega_{\bar{N}} v^4}{\omega_{N_P}^2 \Omega((\omega_N + \omega_{\bar{N}} - 2\omega_{N_P})^2 + 4\Omega^2)}.$$

Lastly, by assuming that, for N (and $\bar{N}$) very close to N_P, we may take $\omega_{N_P} \approx \omega_N \approx \omega_{\bar{N}} \equiv \omega_0$, we arrive at

$$P_{\bar{N}} = (\gamma P_{N_P} L)^2 |F_0|^6 \hbar\omega_0 v/(2L), \qquad (6)$$

where F_0 is an on-resonance ($\delta_N = 0$) field enhancement factor. Comparing with (5), we identify $\frac{\hbar\omega_0 v}{2L|F_0|^2}$ as playing the role of the classical "seed" power in the spontaneous calculation.

In conclusion, we have theoretically studied spontaneous and stimulated FWM in a ring resonator side coupled to a single channel, deriving power scaling relationships for both that should allow the use of experimental results in the classical regime to predict photon-pair generation rates in the quantum regime. More generally, we have derived the biphoton wave function that will result. Considering different pump pulses, we have shown a dramatic variation in the Schmidt number of the biphoton component associated with a single pair of resonances, from near unity for a 5 ps pulse incident on a standard ring [7] to orders of magnitude larger for pulses in the nanosecond regime and longer.

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