Super spontaneous four-wave mixing in single-channel side-coupled integrated spaced sequence of resonator structures

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We consider spontaneous four-wave mixing (SFWM) in a single-channel side-coupled integrated spaced sequence of resonators within a fully quantum formalism. We show that the probability of photon pair production can scale quadratically with the number of resonators, a phenomenon we call super SFWM, in analogy with super-radiant spontaneous emission. Remarkably, in this situation the spectral probability density of the generated photons is independent of the number of rings.

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Quantum correlated photon pair sources continue to get smaller and more intricate. Starting from bulk-crystal optics [1], the catalog of sources has expanded to include Bragg reflection waveguides [2], photonic crystal slab waveguides [3], and microring resonators side-coupled to bus waveguides [4–7]. Yet there are still more exotic structures that have been investigated in the framework of classical nonlinear optics, and only a few of them have been studied for their use in quantum optics (see, e.g., [8]).

In this Letter we focus on a single-channel side-coupled integrated spaced sequence of resonators (SCISSOR) composed of N identical rings of radius $\mathcal{R}$ and quality factor Q, with Λ the distance between two consecutive coupling points [see Fig. 1(a)] [9]. We investigate the biphoton wave function (BWF) of photon pairs generated by spontaneous four-wave mixing (SFWM) in a SCISSOR, demonstrating a simple relation between this expression and that describing SFWM in a single microring resonator side-coupled to a channel waveguide [10].

Following the asymptotic-in/out field formalism of [11] and employing the backward Heisenberg picture approach [12], one can describe SFWM in an arbitrary integrated device. For a generic structure with two ports, left (L) and right (R), taking the pump beam to be incident from the left and the generated photons to exit the structure to the right, the BWF can be written as

$$\phi_{RR;LL}(\omega_1,\omega_2) = \frac{2\sqrt{2}\pi\alpha^2}{\beta}\frac{i}{\hbar}\sqrt{\frac{\omega_1\omega_2}{v_g(\omega_1)v_g(\omega_2)}}\frac{3\hbar^2}{8\epsilon_0} \times \int \mathrm{d}\omega_3\Bigg[\phi_P(\omega_1+\omega_2-\omega_3)\phi_P(\omega_3) \times \sqrt{\frac{\omega_3(\omega_2+\omega_1-\omega_3)}{v_g(\omega_3)v_g(\omega_2-\omega_1-\omega_3)}} \times J_{RR;LL}(\omega_1,\omega_2,\omega_3,\omega_2+\omega_1-\omega_3)\Bigg], \quad (1)$$

where $\phi_P(\omega)$ describes the pump pulse spectrum, $v_g(\omega)$ is the group velocity evaluated at ω, and $|\alpha|^2$ and $|\beta|^2$ are the average number of pump photons and generated pairs per pump pulse in the limit of a low probability of pair production [10], respectively. Here

$$J_{RR;LL}(\omega_1,\omega_2,\omega_3,\omega_4) = \int \mathrm{d}\mathbf{r}\Gamma_3^{ijkl}(\mathbf{r})[D^{i,\text{asy-in}}_{Rk(\omega_1)}(\mathbf{r}) \times D^{j,\text{asy-in}}_{Rk(\omega_2)}(\mathbf{r})D^{k,\text{asy-in}}_{Lk(\omega_3)}(\mathbf{r})D^{l,\text{asy-in}}_{Lk(\omega_4)}(\mathbf{r})] \quad (2)$$

is the overlap integral of the asymptotic-in fields $\mathbf{D}^{\text{asy-in}}_{mk}(\mathbf{r})$, where for simplicity we have considered a single-mode channel, and $\Gamma_3^{ijkl}(\mathbf{r})$ characterizes the material third-order optical nonlinearity, neglecting dispersion and magneto-optic effects, and is simply related to the usual nonlinearity $\chi_3^{ijkl}(\mathbf{r})$ [12].

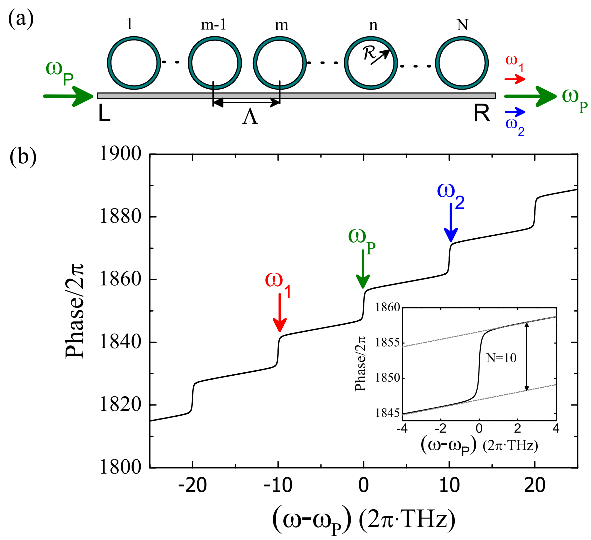

Fig. 1. (Color online) (a) Sketch of a one-channel SCISSOR and (b) transmission phase delay as a function of frequency detuning from ω_P for a SCISSOR composed of $N = 10$ rings with $\mathcal{R} = 10$ μm, $Q = 10^4$, and $\Lambda = 30$ μm. Possible signal and idler frequencies are indicated. Inset: Magnification near ω_P.

Turning now to the specific device discussed above, in the linear regime and when losses can be neglected, the entire SCISSOR is an all-pass filter and the fields $\mathbf{D}_{mk}^{\text{asy-in}}(\mathbf{r})$ at the coupling points of two consecutive rings differ only by a phase factor $e^{i\theta(\omega)}$. In particular, this phase shift at a given ω is the sum of the contribution $\theta_r(\omega)$ given by the transmission through a single ring and the phase shift $k(\omega)\Lambda$ due to the propagation between the two coupling points. Thus we can write

$$J_{RR;LL}(\omega_1,\omega_2,\omega_3,\omega_4) = J_{1,RR;LL}(\omega_1,\omega_2,\omega_3,\omega_4) \times \sum_{m=1}^{N} e^{i\Theta_m(\omega_1,\omega_2,\omega_3,\omega_4)}, \quad (3)$$

where $J_{1,RR;LL}(\omega_1,\omega_2,\omega_3,\omega_4)$ is the field overlap integral for a single ring, and

$$\Theta_m(\omega_1,\omega_2,\omega_3,\omega_4) = \bar{\theta}(\omega_1,\omega_2) + \theta_m(\omega_1,\omega_2,\omega_3,\omega_4), \quad (4)$$

where

$$\bar{\theta}(\omega_1,\omega_2) = (N-1)[\theta_r(\omega_1)+\theta_r(\omega_2)], \quad (5)$$

$$\theta_m(\omega_1,\omega_2,\omega_3,\omega_4) = (m-1)\{\theta_r(\omega_3)+\theta_r(\omega_4)-\theta_r(\omega_1) - \theta_r(\omega_2) + [k(\omega_3)+k(\omega_4) - k(\omega_1)-k(\omega_2)]\Lambda\}, \quad (6)$$

with

$$\theta_r(\omega) = \pi + k(\omega)\mathcal{L} + 2\tan^{-1}\left(\frac{\sigma\sin[k(\omega)\mathcal{L}]}{1-\sigma\cos[k(\omega)\mathcal{L}]}\right). \quad (7)$$

Here $\mathcal{L} = 2\pi\mathcal{R}$ and σ is the self-coupling coefficient describing the single-point coupling between the channel and a ring [10].

We now consider a pulse centered at one ring resonance ω_P (i.e., $k(\omega_P)\mathcal{L} = 2\pi q$, with $q \in \mathbb{Z}$) long enough that we can take $\omega_3 \simeq \omega_P$ and $\omega_1+\omega_2 \simeq 2\omega_P$ in Eq. (1); we take the phase shift for the pump fields as constant [13], and the expression for the BWF becomes

$$\phi_{RR;LL}(\omega_1,\omega_2) = \frac{\beta_1}{\beta}\phi_1(\omega_1,\omega_2)\sum_{m=1}^{N} e^{i\Theta_m(\omega_1,\bar{\omega},\omega_P,\omega_P)}, \quad (8)$$

where $\bar{\omega} \equiv 2\omega_P - \omega_1$, and

$$\phi_1(\omega_1,\omega_2) = \frac{2\sqrt{2}\pi\alpha^2}{\beta_1 v_g(\omega_P)}\frac{i}{\hbar}\sqrt{\frac{\omega_1\omega_2}{v_g(\omega_1)v_g(\omega_2)}}\frac{3\hbar^2}{8\epsilon_0} \times \int \mathrm{d}\omega_3[\phi_P(\omega_1+\omega_2-\omega_3)\phi_P(\omega_3) \times \sqrt{\omega_3(\omega_1+\omega_2-\omega_3)} \times J_{1,RR;LL}(\omega_1,\omega_2,\omega_3,\omega_1+\omega_2-\omega_3)] \quad (9)$$

is the normalized BWF for a single ring, with $|\beta_1|^2$ the corresponding average number of generated pairs. The normalization of Eq. (8) gives the number of photon pairs generated by the SCISSOR:

$$|\beta|^2 = |\beta_1|^2 \int \mathrm{d}\omega_1 \mathrm{d}\omega_2 |\phi_1(\omega_1,\omega_2)|^2 \left|\sum_{m=1}^{N} e^{i\theta_m(\omega_1,\bar{\omega},\omega_P,\omega_P)}\right|^2. \quad (10)$$

When group velocity dispersion (GVD) can be neglected, one can verify that Eq. (6) is independent of m, and thus see that the average number of generated pairs $|\beta|^2 = N^2|\beta_1|^2$ scales quadratically with the number of rings. In the limit of a true CW pump [14], this leads to a rate of entangled photon generation that scales quadratically with the number of rings, in analogy with Dicke superradiance [15], where the emission rate scales quadratically with the number of atoms. The analogy goes beyond a simple superlinear scaling, in that the vacuum fluctuations that are important for the photon generation have to be understood as acting over all the rings as a single quantum system just as they act over all the atoms in Dicke super-radiance, and hence we refer to this phenomenon as super SFWM.

Under these conditions the total BWF is the same (normalized) BWF as for a single microring resonator side-coupled to a channel waveguide,

$$\phi_{RR;LL}(\omega_1,\omega_2) = \phi_1(\omega_1,\omega_2), \quad (11)$$

and is independent of N. Note that in a single-channel SCISSOR there is no mechanism of contradirectional coupling between the rings, and thus, unlike in coupled resonator optical waveguides (CROWs) [8], the super SFWM is not due to the formation of photonic bands.

An interesting problem is the effect of a finite GVD. In the telecom range, guided mode dispersion of silicon nanowires can be easily tuned, with GVD from positive to negative [2000 to −1000 ps/(nm · km)], by changing the cross-sectional shape of the waveguide [16]. Thus, we consider the limit of small dispersion, with

$$k(\omega) \simeq k(\omega_P) + \frac{1}{v}(\omega-\omega_P) + \frac{1}{2}\xi(\omega-\omega_P)^2, \quad (12)$$

where v and ξ are the group velocity and the GVD parameter evaluated at ω_P, respectively. A finite GVD implies that generated photon frequencies spaced equally on opposite sides of the pump frequency ω_P do not have corresponding wave vectors spaced equally on opposite sides of the pump wave vector $k(\omega_P)$, since

$$\delta k \equiv 2k(\omega_P) - k(\omega_1) - k(\bar{\omega}) = -\xi(\omega_1-\omega_P)^2, \quad (13)$$

so that

$$\theta_m(\omega_1,\bar{\omega},\omega_P,\omega_P) \simeq (m-1)\delta k\left(\Lambda + \mathcal{L} + \frac{2\mathcal{L}\sigma}{1-\sigma}\right), \quad (14)$$

where the first term in the final parentheses is due to the wave vector mismatch between pump and generated fields as they propagate between rings, and the last two terms are due to the transmission through a single ring, i.e., using Eqs. (12) and (13) in Eq. (7). In general,

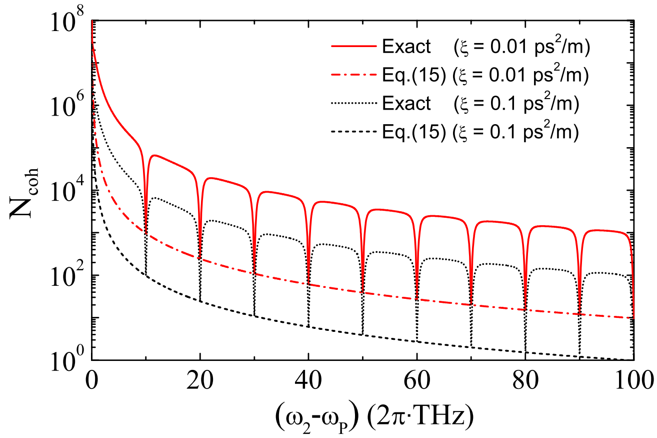

Fig. 2. (Color online) Coherence number as a function of the frequency difference between the pump and high energy generated photons for triply resonant generation. We consider $R = 10$ μm, $Q = 10^4$, and take $\omega_P = 2\pi \cdot 200$ THz, $v \simeq 10^8$ ms^{-1}, $\xi \simeq 0.01$ ps^2/m.

for high-Q resonators ($\sigma \simeq 1$), the final term in Eq. (14) dominates, and we can define a *coherence number* similar to the usual coherence length $L_{\mathrm{coh}} = \pi/|\delta k|$:

$$N_{\mathrm{coh}} = \left\lfloor \frac{\pi}{|\theta_m/(m-1)|} \right\rfloor \simeq \left\lfloor \frac{\pi \omega_P}{4Qv|\delta k|} \right\rfloor, \qquad (15)$$

where $\lfloor x \rfloor$ is the floor function and $Q \simeq \omega_P L/[2(1-\sigma)v]$ is the quality factor of the ring. The coherence number is the maximum number of rings for which the generation efficiency scales with N^2. It is inversely proportional to Q, as well as the GVD parameter and the spectral distance of the two resonances on which we consider the SFWM process [see Eq. (13)]. As expected, this number diverges when $|\xi| \to 0$.

If we consider a silicon ring with representative values [16] $\mathcal{R} = 10$ μm, $Q = 10^4$, and take $\omega_P = 2\pi \cdot 200$ THz, $v \simeq 10^8$ ms^{-1}, $\xi \simeq 0.01$ ps^2/m [−8 ps/(nm · km)], we find $N_{\mathrm{coh}} \simeq 1000$ for $\omega_P - \omega_1 \simeq 2\pi\mathrm{FSR} = c/(n\mathcal{R})$ (i.e., the nearest neighbor resonances). In particular, in Fig. 2, we plot the coherence number given by Eq. (15) as a function of the frequency difference between the pump and high energy generated photons. We also show the curve corresponding to the coherence number calculated considering the exact phase factor given by Eq. (6). As expected, the two curves agree best when photons are generated near a ring resonance, which corresponds to the largest generation rate [10], with the approximate expression [Eq. (15)] providing a lower bound. So while the inclusion of GVD invalidates Eq. (11), the quadratic scaling of the rate of generated photons with the number of rings remains approximately valid for up to several hundreds of rings in an ideal SCISSOR when the idler and signal are sufficiently close to the pump frequency. Of course, in a real system, fabrication imperfections might cause the misalignment of ring resonances and prevent quadratic scaling from continuing for hundreds of rings. However, recently SFWM in a 35-ring CROW structure was demonstrated [8], as were pair generation rates of several megahertz in a single resonator [5]. Thus even for 35 rings we expect that pair generation rates of several megahertz could be achieved in a SCISSOR, but with a reduction in pump power of over three orders of magnitude compared to [5].

In conclusion, we have derived very general expressions for both the rate of SFWM in a single-channel SCISSOR and the BWF that describes the entangled photons. We have shown that in the limit of negligible GVD the pair generation rate scales superlinearly with the number of rings N, while the BWF is independent of N. We have generalized these results to include the effects of small GVD.

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