Stimulated Emission Tomography

M. Liscidini[1] and J. E. Sipe[2]

[1]Dipartimento di Fisica, Università degli Studi di Pavia, via Bassi 6, I-27100 Pavia, Italy

[2]Department of Physics and Institute for Optical Sciences, University of Toronto, 60 St. George Street, Ontario M5S 1A7, Canada

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We identify a relation between the number of photon pairs generated by parametric fluorescence, through either spontaneous parametric down-conversion (SPDC) or spontaneous four-wave mixing, and the number generated by the corresponding stimulated process, respectively, either difference-frequency generation or stimulated four-wave mixing. On the basis of this very general result, we show that the characterization of SPDC sources of two-photon states in a given system can be performed solely by studying stimulated emission. We call this technique stimulated emission tomography (SET). We show that the number of photons detected in SET can be 9 orders of magnitude larger than the average number of coincidence counts in two-photon quantum state tomography. These results open the way to the study of sources of quantum-correlated photon pairs with unprecedented precision and unparalleled resolution.

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The birth of quantum optics can be identified with Einstein's introduction of his coefficients linking absorption, stimulated emission, and spontaneous emission [1]. The relation between stimulated emission, many aspects of which can be understood classically, and spontaneous emission, which it is generally agreed cannot be understood classically, is particularly striking. Arising because the same quantum electromagnetic field is responsible for both these processes [2], it is central to quantum electrodynamics. While one could imagine there being theoretical errors in calculating the spontaneous or stimulated emission rate for a particular transition, or experimental errors in determining them in the laboratory, a wholesale violation of the link between spontaneous and stimulated emission is conceivable only with the supposition that our current quantum mechanical description of matter and radiation is fundamentally flawed.

In quantum information processing, the success or security of various tasks is based on the presumed correctness of our current quantum mechanical description; one example is the security of quantum key distribution [3]. It is thus natural to ask if the link between stimulated and spontaneous processes provided by that quantum mechanical description can be used to facilitate or simplify any of the tasks of quantum information processing. Such a simplification would base its correctness only on the authority of our quantum mechanical description of the interaction of matter and radiation.

In this Letter, we argue that such a simplification is possible, not utilizing the link between spontaneous and stimulated emission, but rather the corresponding link between the spontaneous processes leading to the production of quantum-correlated pairs of photons and their stimulated analogues. The task that we claim can be simplified, and vastly so, is the characterization of the biphoton wave function that describes those quantum-correlated pairs.

Quantum-correlated pairs of photons are generated in nonlinear crystals either by spontaneous four-wave mixing, in which two pump photons spontaneously convert to a signal and idler photon, typically at different energies, or spontaneous parametric down-conversion (SPDC), in which a pump photon spontaneously fissions into a signal and idler photon, respectively, at energies typically slightly greater than half and slightly less than half of the pump photon energy [4–7]. In this Letter, we focus on SPDC for simplicity, but the discussion can be easily generalized to spontaneous four-wave mixing. Often, the photons are polarization entangled, and the rate of pair production is usually low, both because of the weakness of the nonlinearity and because it would be deleterious for applications if the pump pulse were so strong that multiple pairs could be produced in its time window. The polarization state of a pair that is occasionally generated by a pump pulse can be determined using the techniques of quantum state tomography [8].

We show below that this task can be greatly simplified by using the stimulated version of SPDC, difference-frequency generation (DFG), in which a seed signal pulse is injected together with a pump pulse. By varying the properties of the input seed signal pulse so that the output signal pulse ranges through the characteristics that a spontaneously generated signal pulse might have in the absence of the seed, and measuring the large idler output in the presence of the seed, we can extract the information necessary to identify the biphoton wave function generated when a pair is produced in the absence of the seed. Thus, by the use of a seed signal pulse in addition to the pump pulse, we can perform stimulated emission tomography of the polarization state that would be produced in the absence of the seed. While the availability of a source of seed pulses is an additional experimental requirement, we demonstrate that the savings in time and improvement in precision are so great that this should become the preferred method for characterizing sources of two-photon states generated by parametric fluorescence.

We consider the general situation sketched in Fig. 1(a), in which photon pairs are generated by SPDC in a generic nonlinear (NL) structure. The effect of the nonlinearity in generating the down-converted photons can be described by a formal transition at $t = 0$ from an asymptotic-in to an asymptotic-out state, with these states otherwise involving only linear propagation, however complicated [9]. If the probability of generating photons in each pair of modes $(\nu \mathbf{k}_1, \eta \mathbf{k}_2)$ is sufficiently small, then if a pair is generated, the state describing the down-converted photons can be written as [10]

$$|II\rangle = \frac{1}{\sqrt{2}} \sum_{\nu,\eta} \int d\mathbf{k}_1 d\mathbf{k}_2 \phi_{\nu\eta}(\mathbf{k}_1, \mathbf{k}_2) b^\dagger_{\nu\mathbf{k}_1} b^\dagger_{\eta\mathbf{k}_2} |\mathrm{vac}\rangle, \quad (1)$$

(see the Supplemental Material [11]) where $\phi_{\nu\eta}(\mathbf{k}_1, \mathbf{k}_2)$ is the normalized biphoton wave function, ν and η indicate the polarization of the generated photons, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors, and $b^\dagger_{\nu\mathbf{k}_1}$ and $b^\dagger_{\eta\mathbf{k}_2}$ are the corresponding creation operators.

A common experiment to characterize two-photon sources is a set of coincidence measurements in a configuration similar to the one shown in Fig. 1(b), in which two single-photon detectors count the photon pairs generated at

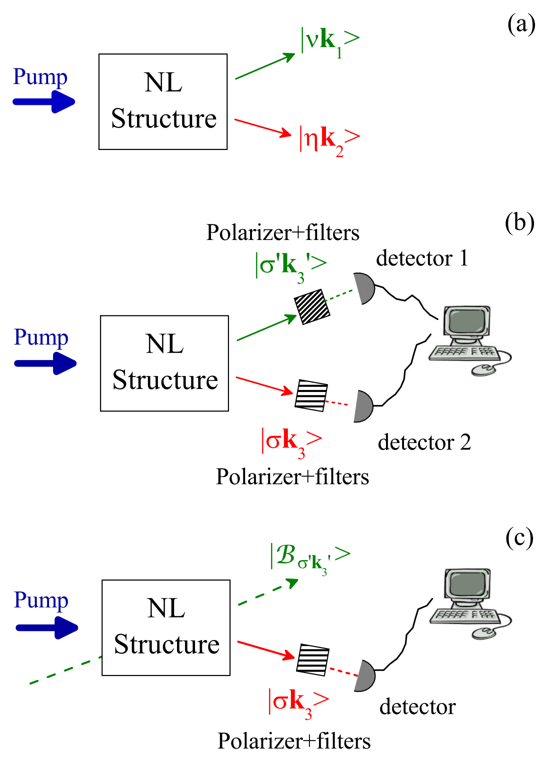

FIG. 1 (color online). (a) Emission of two photons by parametric fluorescence. (b) Sketch of the setup for the polarization resolved coincidence measurement. (c) Corresponding stimulated emission configuration.

wave vectors in the neighborhood of $\mathbf{k}_3$ and $\mathbf{k}'_3$, and with specified polarizations σ and σ'. Here, we consider configurations in which the generated photons are either noncollinear or have different energies, such that they can always be separated; thus, we always have $\mathbf{k}'_3 \neq \mathbf{k}_3$. If one wants to reconstruct the biphoton wave function, the detection is done by taking small ranges $\delta\mathbf{k}_3$ and $\delta\mathbf{k}'_3$ over which variation of the biphoton wave function can be considered negligible. In practice, in free space, such a situation can be realized using two pinholes, which restrict the light in the directions transverse to the propagation direction towards the detector. For single-mode optical fibers or channels in integrated devices, such spatial filtering is not necessary, as the transverse profile of spontaneous emission is determined by the mode profile of the waveguide. The final restriction, in the length of the wave vector, is done with optical filters. In any case, the average number of generated pairs in the specified ranges is easily found to be

$$\langle b^\dagger_{\sigma\mathbf{k}_3} b_{\sigma\mathbf{k}_3} b^\dagger_{\sigma'\mathbf{k}'_3} b_{\sigma'\mathbf{k}'_3} \rangle \delta\mathbf{k}_3 \delta\mathbf{k}'_3 = 2|\gamma|^2 |\phi_{\sigma,\sigma'}(\mathbf{k}_3, \mathbf{k}'_3)|^2 \delta\mathbf{k}_3 \delta\mathbf{k}'_3, \quad (2)$$

where $|\gamma|^2$ is the probability that a pair of photons is generated.

Using the same quantum theoretical approach, we now consider the scenario shown in Fig. 1(c), in which a proper combination of incoming beams results in a coherent state exiting from the structure with polarization σ' and centered at $\mathbf{k}'_3$. The asymptotic-out state satisfies

$$B_{\sigma'\mathbf{k}'_3} |\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle = \mathcal{B}_{\sigma'\mathbf{k}'_3} |\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle, \quad (3)$$

where $|\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2$ is the average number of photons in the coherent state, and

$$B^\dagger_{\sigma'\mathbf{k}'_3} = \int d\mathbf{k} f_{\sigma'\mathbf{k}'_3}(\mathbf{k}) b^\dagger_{\sigma'\mathbf{k}} \quad (4)$$

with $f_{\sigma'\mathbf{k}'_3}(\mathbf{k})$ a normalized function centered at $\mathbf{k}'_3$. For bulk crystals [6], optical fibers [12], and one-channel integrated optical devices [13], the coherent state $|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle$ can be realized with a single input beam. For more complicated structures, the construction of $|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle$ might require a combination of more than one input beam, but it can always be obtained [9].

We consider the number of photons exiting from the structure at $\mathbf{k}_3$ with polarization σ [see Fig. 1(c)]; these photons are generated by parametric fluorescence as well as by the corresponding nonlinear process stimulated by the coherent state, i.e., DFG [14]. In the limit of an undepleted pump, the average number of generated photons is

$$\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}}\rangle_{|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle}\delta\mathbf{k_3} \approx 2|\gamma|^2\sum_\eta \int d\mathbf{k}|\phi_{\sigma\eta}(\mathbf{k_3},\mathbf{k})|^2\delta\mathbf{k_3} + 2|\gamma|^2|\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2\int d\mathbf{k_1}d\mathbf{k_2} \times \phi_{\sigma\sigma'}(\mathbf{k_3},\mathbf{k_1})\phi^*_{\sigma\sigma'}(\mathbf{k_3},\mathbf{k_2}) \times f^*_{\sigma'\mathbf{k}'_3}(\mathbf{k_1})f_{\sigma'\mathbf{k}'_3}(\mathbf{k_2})\delta\mathbf{k_3}, \tag{5}$$

which, neglecting the contribution of spontaneous emission (the first term on the right-hand side) and considering a stimulating beam narrowly centered at $\mathbf{k}'_3$, becomes

$$\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}}\rangle_{|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle}\delta\mathbf{k_3} \approx 2|\gamma|^2|\phi_{\sigma\sigma'}(\mathbf{k_3},\mathbf{k}'_3)|^2 \times \frac{(2\pi)^3\mathcal{S}(\mathbf{r}_0)}{c\hbar\omega'_3}\delta\mathbf{k_3}, \tag{6}$$

where $\omega'_3 = ck'_3$ and

$$\mathcal{S}(\mathbf{r}_0) = \frac{c\hbar\omega'_3}{8\pi^3}|\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2\left|\int d\mathbf{k} f_{\sigma'\mathbf{k}'_3}(\mathbf{k})e^{-i(\mathbf{k}-\mathbf{k}'_3)\cdot\mathbf{r}_0}\right|^2 \tag{7}$$

is the modulus of the Poynting vector at position $\mathbf{r}_0$ at $t = 0$ of a nominal field that, propagating exclusively in vacuum, would evolve into the outgoing seed pulse; $\mathbf{r}_o$ is determined by properties of the source, *inter alia* its position (see the Supplemental Material [11]). To see the significance of this result, we consider a seed pulse sufficiently centered at $\mathbf{k}'_3$ that $\mathcal{S}(\mathbf{r}_0)$ can be taken to be the value $\mathcal{S}$ at the center of the nominal seed pulse. If we now assume that both the range $\delta\mathbf{k}'_3$ of the nominal seed pulse and the range $\delta\mathbf{k_3}$ identifying the collection interval of the stimulated photons are restricted to the respective ranges employed in the spontaneous experiment, then by using Eq. (2) in Eq. (6), we obtain

$$\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}}\rangle_{|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle}\delta\mathbf{k_3} \approx \langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}} b^\dagger_{\sigma'\mathbf{k}'_3} b_{\sigma'\mathbf{k}'_3}\rangle\delta\mathbf{k_3}\delta\mathbf{k}'_3 \times \frac{(2\pi)^3}{\delta\mathbf{k}'_3}\frac{\mathcal{S}}{c\hbar\omega'_3}, \tag{8}$$

which means that the expected number of photons stimulated into $\delta\mathbf{k_3}$ is proportional to the expected number of pairs, one each into $\delta\mathbf{k_3}$ and $\delta\mathbf{k}'_3$, that would be generated in the absence of stimulation. Note that in experiments with single-mode optical fibers or channel waveguides, the restriction mentioned above would be particularly easy to satisfy; only optical filtering to set the frequency range would be required because the transverse profile of the fields would be set by the waveguide modes.

The physics of Eq. (8) can be made more explicit by remembering that since the stimulating beam is strongly peaked around k'_3, we can write

$$\frac{(2\pi)^3}{\delta\mathbf{k}'_3}\frac{\mathcal{S}}{c\hbar\omega'_3} \approx V\frac{\mathcal{S}}{c}\frac{1}{\hbar\omega'_3} \approx |\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2, \tag{9}$$

where $V = (2\pi)^3/\delta\mathbf{k}'_3$ can be taken as the volume of the nominal stimulating pulse in the direct space and $\mathcal{S}/c$ the corresponding energy density. Here, the use of $\approx$ refers to the approximation that the energy density can be taken constant in the entire volume V; a more precise relation would involve the shape of the pulse. Finally, using Eq. (9) in Eq. (8), we obtain

$$\frac{\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}}\rangle_{|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle}\delta\mathbf{k_3}}{\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}} b^\dagger_{\sigma'\mathbf{k}'_3} b_{\sigma'\mathbf{k}'_3}\rangle\delta\mathbf{k_3}\delta\mathbf{k}'_3} \approx |\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2, \tag{10}$$

which states that the ratio between the average number of stimulated photon pairs and those generated by parametric fluorescence is essentially the average number $|\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2$ of photons in the stimulating pulse.

The expression (10) is analogous to the well-known result for stimulated and spontaneous single-photon emission in a two-level system [15]. Of course, there are differences: In the stimulated process we consider here, we have the emission of photon pairs in which one of the photons, and only one, shares the properties of those in the stimulating pulse. And, in the spontaneous process of interest here, the emitted photons can have quantum properties for which no simple analog exists for single-photon emission, such as entanglement.

Spontaneous parametric processes are the most utilized for the generation of polarization-entangled photon pairs [6,7]. The study and characterization of these sources is usually done by means of quantum state tomography, the goal of which is the reconstruction of the density matrix. For example, in the case of polarization-entangled photon pairs, we deal with a matrix in the 2×2 polarization Hilbert space, where $|HH\rangle$, $|VH\rangle$, $|HV\rangle$, and $|VV\rangle$ are usually taken as the basis; V and H stand for vertical and horizontal photon polarization, respectively. The reconstruction of the density matrix can be done by considering the setup sketched in Fig. 1(b); the $4^2 - 1$ independent elements of the matrix are found by means of a set of coincidence measurements spanning σ and σ' over a number of complete four-element bases [8]. If we collect photons in the neighborhood of $\mathbf{k_3}$ and $\mathbf{k}'_3$, and with polarization σ and σ', respectively, the average number of coincidence counts is

$$n_{\sigma\sigma'} = \mathcal{N}^2\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}} b^\dagger_{\sigma'\mathbf{k}'_3} b_{\sigma'\mathbf{k}'_3}\rangle\delta\mathbf{k_3}\delta\mathbf{k}'_3, \tag{11}$$

where $\mathcal{N}$ is the probability of detecting a signal or idler generated photon, which depends on the detectors' efficiency. The general result (10) suggests that $n_{\sigma\sigma'}$ could be obtained from a stimulated experiment as the one shown in Fig. 1(c) by measuring the average number of stimulated idler photons $n_\sigma(\sigma') = \mathcal{N}\langle b^\dagger_{\sigma\mathbf{k_3}} b_{\sigma\mathbf{k_3}}\rangle_{|\mathcal{B}_{\sigma'\mathbf{k}'_3}\rangle}\delta\mathbf{k_3}$, where

$$n_\sigma(\sigma') = \frac{|\mathcal{B}_{\sigma'\mathbf{k}'_3}|^2}{\mathcal{N}}n_{\sigma\sigma'}. \tag{12}$$

So, by measuring $n_{\sigma}(\sigma')$, we perform a kind of virtual tomography on the state that would be emitted by the corresponding spontaneous process. We call this stimulated emission tomography (SET).

It is worth noticing that the experiment in Fig. 1(c) corresponds to DFG, which can be described in the framework of classical electromagnetic theory for a sufficiently large $|\mathcal{B}_{\sigma'\mathbf{k}_3'}|^2$. Thus, the result (12) demonstrates that the density matrix describing the two-photon state generated by parametric fluorescence can be reconstructed through a "classical" experiment. Although we have limited our discussion to the determination of the polarization state of the photon pairs, the entire biphoton wave function (i.e., modulus and phase) associated with the spontaneous generation of photon pairs acts as the response function characterizing how a seed pulse stimulates the emission of photons [see Eq. (8) in the Supplemental Material [11]]. Thus, it should be possible to design similar classical experiments to determine the entire biphoton wave function.

The use of SET for the reconstruction of the density matrix also has a number of practical advantages. The most important follows directly from Eq. (12), which shows that $n_{\sigma}(\sigma')$ is proportional to the number of stimulating photons, and thus it can be several orders of magnitude larger than $n_{\sigma\sigma'}$ measured in two-photon quantum state tomography. Moreover, while $n_{\sigma\sigma'}$ is evaluated by performing a large ensemble of measurements, for a sufficiently large $|\mathcal{B}_{\sigma'\mathbf{k}_3'}|^2$, the number of stimulated pairs generated per pulse is essentially constant, so $n_{\sigma}(\sigma')$ can be obtained by a single measurement, and without the need for single-photon detectors. To give a quantitative idea of the benefits of SET, we consider the pioneering experiment of Kwiat *et al.* [6], in which photons are generated by cw SPDC with 150 mW of pump power. The cw pump beam can be viewed as a sequence of uncorrelated pulses of duration equal to the coherence time of the laser, which can be safely taken to be about 10 μs (corresponding to a laser linewidth of about 100 kHz). In a SET experiment, we can consider a stimulating pulse of the same duration and a power of 1 mW, which guarantees the validity of the undepleted pump approximation. Assuming 100% of collection efficiency (i.e., $\mathcal{N} = 1$), this corresponds to a number of stimulated idler counts per pulse about 10^9 the average number of coincidence counts per pulse in the corresponding tomography experiment.

A second important advantage comes from the fact that in two-photon quantum state tomography, the resolution in $\mathbf{k}_3$ and $\mathbf{k}_3'$ is limited by the low generation rate and by the detector sensitivity, as it is obtained by means of spectral and/or spatial filters. In SET, the number of idler counts is very large, and this gives the possibility of performing an experiment with unparalleled resolution, wherein $\mathbf{k}_3'$ is essentially limited by the laser linewidth. This is particularly interesting not only for a precise reconstruction of the density matrix but also to obtain high-definition measurements of the energy joint spectral probability density of the photon pairs [16]. The approach based on stimulated emission is especially interesting for characterizing sources in which high-Q resonators are used to enhance the material nonlinearity, and thus where high energy resolution is required [17].

In conclusion, we demonstrated a simple relation between spontaneous and stimulated emission of photon pairs. Starting from this fundamental and general result, we showed that the characterization of sources of two-photon states generated by parametric fluorescence can be performed much faster than in two-photon quantum state tomography, with unprecedented precision and unparalleled resolution.

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