Disorder-induced losses in photonic crystal waveguides with line defects

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A numerical analysis of extrinsic diffraction losses in two-dimensional photonic crystal slabs with line defects is reported. To model disorder, a Gaussian distribution of hole radii in the triangular lattice of airholes is assumed. The extrinsic losses below the light line increase quadratically with the disorder parameter, decrease slightly with increasing core thickness, and depend weakly on the hole radius. For typical values of the disorder parameter the calculated loss values of guided modes below the light line compare favorably with available experimental results. © 2004 Optical Society of America

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Linear waveguides in photonic crystal (PhC) slabs are attractive candidates for the realization of integrated optical interconnects and other photonic devices.[1] Propagation of light in these systems is controlled by a photonic lattice in the two-dimensional plane and by the dielectric discontinuity of the slab in the vertical direction. In PhC slabs with a strong refractive-index contrast, such as a suspended membrane or air bridge and a silicon-on-insulator (SOI) system, the defect mode associated with the linear waveguide may lie partly below the cladding light line in the $k-\omega$ plane.[2,3] In this case the mode is truly guided with no intrinsic diffraction loss (unlike modes above the light line, which are subject to intrinsic out-of-plane losses). In such a situation the propagation losses of a guided mode depend exclusively on the presence of structural imperfections. The role of disorder in propagation losses is therefore a crucial issue for prospective applications of linear PhC waveguides to integrated optics.

In this work we present a theoretical treatment of the effect of disorder on propagation losses in two-dimensional PhC slabs with line defects. We focus on the most commonly studied structure, namely, a missing row of holes along the ΓK direction in a triangular lattice of airholes, which is also called a W1 waveguide. The main purpose of this Letter is to quantify the trends of disorder-induced losses as a function of various structure parameters and to make a comparison between air-bridge and SOI PhC slabs.

Previous studies of disorder phenomena in PhC slabs concentrated on the effects on the photonic gap[4] or on the consequences of a nonvertical shape of the holes for propagation losses.[5] Recently, the effects of scattering at sidewall roughness in PhC slabs were studied in a two-dimensional model.[6] In this work we consider the variation of hole radii as the main disorder effect. The present fully three-dimensional results extend those of Ref. 7, in which only intrinsic losses above the light line were calculated and the effect of disorder was not included. After discussing the model and the numerical results, we compare them with recent experimental measurements of losses below the light line.[8,9]

We consider a slab of dielectric constant ϵ_{core} and thickness d along z, patterned with a triangular lattice of airholes in the slab plane (xy), with lattice constant a, hole radius r, and a line defect with channel width w [see inset in Fig. 1(a)]; for a W1 waveguide, $w = w_0 = \sqrt{3}\,a$. The photonic eigenmodes are calculated by a recently developed approach consisting of an expansion in the basis of guided modes of an effective homogeneous waveguide.[10] Coupling to radiative modes of the effective waveguide is taken into account by time-dependent perturbation theory, which yields the imaginary part of the mode frequency $\text{Im}(\omega)$.[11,12] To take into account the effects of disorder, we define a large supercell in the direction parallel to the line defect, in which the hole radii are randomly distributed with Gaussian probability around an average value r. The rms deviation Δr of the radius is taken as the disorder parameter. The random variation of the hole size changes the dielectric modulation to $\epsilon_{\text{dis}}(\mathbf{r})$ and gives rise to a perturbation $\Delta\epsilon^{-1}(\mathbf{r}) = \epsilon_{\text{dis}}^{-1}(\mathbf{r}) - \epsilon^{-1}(\mathbf{r})$, which couples guided and radiative eigenmodes and leads to a finite

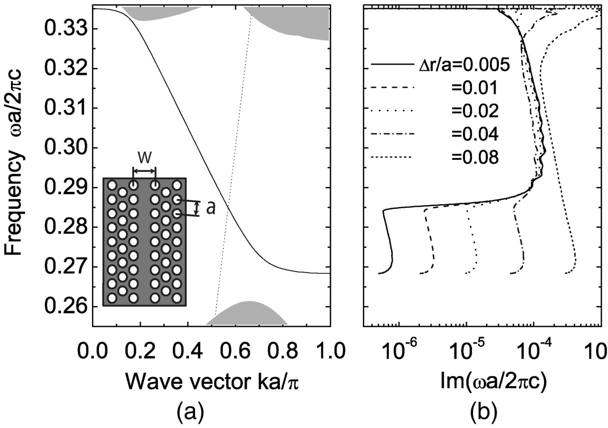

Fig. 1. (a) Dispersion of the defect mode; the dotted line is the air light line. (b) Imaginary part of the frequency for different values of disorder parameter $\Delta r/a$. Parameters of the W1 air-bridge structure are $r/a = 0.28$, $d/a = 0.5$, $\epsilon_{\text{core}} = 12$, $\epsilon_{\text{air}} = 1$.

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$\mathrm{Im}(\omega)$ also for truly guided modes.[13] The propagation losses are defined as $\alpha = 2\,\mathrm{Im}(k) = 2\,\mathrm{Im}(\omega)/v_g$, where $v_g = \mathrm{d}\omega/\mathrm{d}k$ is the group velocity.

In Fig. 1 we consider a W1 waveguide in the air bridge and show the dispersion of the defect mode and the imaginary part of the frequency for different values of $\Delta r/a$. Only the defect mode that is even with respect to the horizontal midplane ($\sigma_{xy} = +1$) and odd with respect to the plane of incidence ($\sigma_{kz} = -1$) is considered.[14] The shaded regions in Fig. 1(a) represent the bulk PhC modes. As can be seen from Fig. 1(b), the defect mode is subject mainly to intrinsic losses when its dispersion falls above the air light line.[7] When the mode crosses the light line, $\mathrm{Im}(\omega a/2\pi c)$ has a sudden decrease toward finite values, due to the disorder-induced losses. The imaginary part of the mode frequency depends strongly on the disorder parameter of the structure. In particular, it is found to grow almost quadratically with Δr, as is appropriate for a Rayleigh-scattering mechanism. The extrinsic losses become important also above the light line for sufficiently high values of the disorder parameter or in the energy window of low group velocity, i.e., for $\omega a/2\pi c > 0.33$ in Fig. 1.

In Fig. 2 the dependence of $\mathrm{Im}(\omega a/2\pi c)$ on various structure parameters is displayed for $\Delta r/a = 0.01$. We show $\mathrm{Im}(\omega a/2\pi c)$ with decreasing values of the core thickness [Fig. 2(a)] and increasing average hole radius [Fig. 2(b)]. Concerning the dependence on the core thickness, it is seen from Fig. 2(a) that both intrinsic and extrinsic losses increase with decreasing d/a due to increased leakout of the defect mode in the patterned regions when the vertical confinement of the field is higher. Below the light line, $\mathrm{Im}(\omega a/2\pi c)$ is between 10^{-6} and 10^{-5}. When we set $d/a = 0.5$ and study the same quantity as a function of the air fraction, we see [Fig. 2(b)] that the intrinsic losses above the light line increase with increasing hole radius, as previously reported,[7] whereas the extrinsic losses are $\sim 3 \times 10^{-6}$ and have a much-weaker dependence on this parameter.

In Fig. 3 we show a comparative analysis of the mode dispersion, group velocity, imaginary part of the frequency, and losses below the light line both in air-bridge and SOI configurations for channel widths of $w = w_0$ (W1) and $w = 0.7w_0$ (W07 waveguide). For SOI systems the reflection through plane xy is not a symmetry operation, and thus both parities must be considered in the basis set.[12] Nevertheless, there is a region below the light line in the $k-\omega$ plane where no parity mixing occurs.[5,15,16] We can see from Fig. 3 that $\mathrm{Im}(\omega a/2\pi c)$ is generally larger in SOI systems, due to the asymmetric slab configuration. Looking at the dependence on channel width, the out-of-plane losses are lower for W1 than for W07 in air-bridge structures, while in SOI the W07 waveguide has lower losses owing to the larger group velocity of the mode at the crossing point with the $\mathrm{SiO_2}$ light line.[15] This explains the experimental situation according to

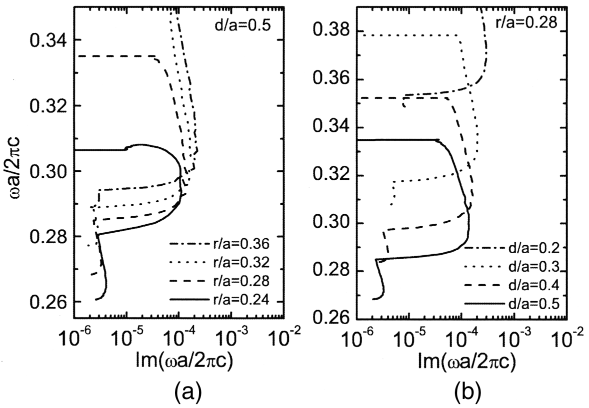


Fig. 2. (a) Imaginary part of the frequency for different core thicknesses of a W1 air bridge with $r/a = 0.28$. (b) Imaginary part of the frequency for different hole radii of a W1 air bridge with $d/a = 0.5$. The disorder parameter is set equal to the typical value of $\Delta r/a = 0.01$.

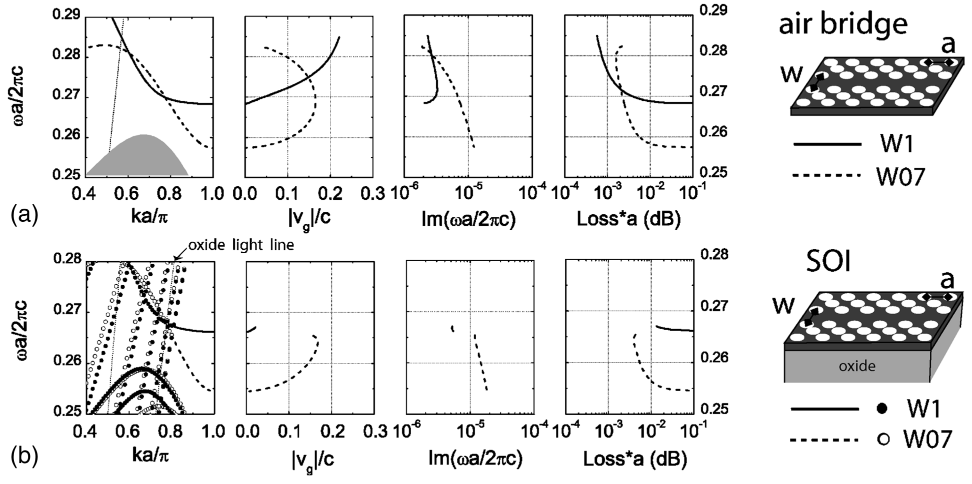
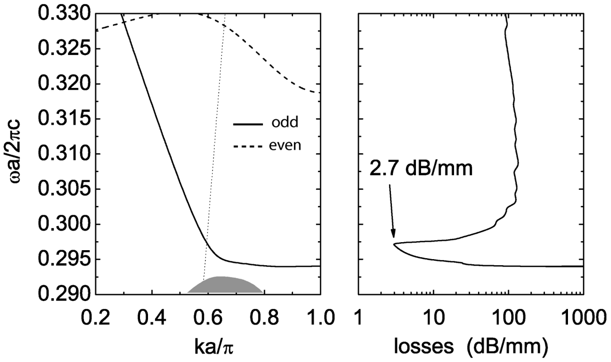


Fig. 3. Comparison between W1 (solid curves and filled circles) and W07 (dashed curves and open circles) linear waveguides in membrane and SOI structures, respectively. (a) Dispersion of the defect mode, group velocity, imaginary part of the frequency, and losses (dimensionless losses, αa in decibels) for an air-bridge PhC waveguide with $r/a = 0.28$, $d/a = 0.5$, $\Delta r/a = 0.01$; (b) same quantities for a SOI-based PhC waveguide with identical structure parameters and $\epsilon_{\mathrm{ox}} = 2.1$ for the $\mathrm{SiO_2}$ substrate. The results for the group velocity, imaginary part of the frequency, and losses are plotted in only the energy range for which the defect mode lies below the light line (or below the parity mixing region for SOI).



(a) (b)

Fig. 4. (a) Defect mode dispersion and (b) propagation losses for the silicon membrane structure studied in Ref. 9: W1 waveguide, $a = 445$ nm, $r/a = 0.37$, $d/a = 0.5$, nominal disorder parameter $\Delta r = 5$ nm, $\epsilon_{\mathrm{Si}} = 12$.

which W1 waveguides are commonly addressed in silicon membranes,[9] whereas W07 waveguides have lower propagation losses in SOI structures.[8] The parameters used in these calculations are close to those of the SOI structures fabricated and studied in Ref. 8. With $a = 390$ nm and $\Delta r \simeq 4$ nm the present results [minimum loss of ~9 dB/mm for W07 SOI waveguides; see last panel in Fig. 3(b)] agree well with the experimental value of 6 dB/mm.[8]

Finally, in Fig. 4 we show a comparison between the present theoretical model and recent experimental results obtained on silicon membranes.[9] The quoted sample parameters were used in the calculations (see caption). In Fig. 4(a), $\sigma_{xy} = +1$ modes, both odd ($\sigma_{kz} = -1$) and even ($\sigma_{kz} = +1$) with respect to the plane of incidence, are shown for completeness. Propagation losses of 24 ± 2.4 dB/cm were measured in Ref. 9 for the $\sigma_{xy} = +1$, $\sigma_{kz} = -1$ defect mode below the light line. In Fig. 4(b) the calculated losses have a value of 27 dB/cm when the corresponding mode crosses the light line, which is in close agreement with the experimental data. Good agreement is also found for the spectral dependence of the losses.

We did not consider disorder-induced scattering into the counterpropagating defect mode.[17] The fair agreement between calculated and measured values for the losses below the light line suggests that out-of-plane scattering into the leaky waveguide modes is indeed the dominant loss mechanism for the high-contrast PhC slabs considered in this work.

The present method is a fast and accurate tool for calculating disorder-induced diffraction losses out of the slab plane in linear PhC waveguides. The trends as a function of various structure parameters have been determined within a model of variable hole radii in a large supercell. Quantitative comparison with experimental results for the losses of truly guided modes is satisfactory when state-of-the-art disorder parameters are assumed.

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References

1. See papers in Feature on *Photonic Crystal Structures and Applications*, T. F. Krauss and T. Baba, eds., IEEE J. Quantum Electron. **38,** 724–963 (2002).
2. A. Chutinan and S. Noda, Phys. Rev. B **62,** 4488 (2000).
3. S. G. Johnson, P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, Phys. Rev. B **62,** 8212 (2000).
4. H.-Y. Ryu, J.-K. Hwang, and Y.-H. Lee, Phys. Rev. B **59,** 5463 (1999).
5. Y. Tanaka, T. Asano, Y. Akahane, B.-S. Song, and S. Noda, Appl. Phys. Lett. **82,** 1661 (2003).
6. W. Bogaerts, P. Bienstman, and R. Baets, Opt. Lett. **28,** 689 (2003).
7. L. C. Andreani and M. Agio, Appl. Phys. Lett. **82,** 2011 (2003).
8. M. Notomi, A. Shinya, K. Yamada, J. Takahashi, C. Takahashi, and I. Yokohama, in Ref. 1, p. 736.
9. S. J. McNab, N. Moll, and Yu. Vlasov, Opt. Express **11,** 2927 (2003), http://www.opticsexpress.org.
10. L. C. Andreani and M. Agio, in Ref. 1, p. 891.
11. L. C. Andreani, Phys. Status Solidi B **234,** 139 (2002).
12. We typically use up to 461 plane waves and two guided modes in the basis set for air-bridge structures, taking advantage of the horizontal mirror symmetry of the slab. The number of guided modes is doubled in the case of SOI. The calculations employ a supercell in the direction ΓM perpendicular to the line defect, and an average over the results with supercell widths from $3w_0 + w$ to $8w_0 + w$ is taken to smooth out finite supercell effects, as in Ref. 7.
13. The supercell along ΓK used to model the disorder typically has a size of $39a$. Note that the use of this supercell does not require the number of plane waves to be increased, since disorder-induced scattering is treated by perturbation theory. All loss results include an average over calculations with six different random distributions corresponding to the same disorder parameter $\Delta r/a$.
14. The defect mode shown in Fig. 1 is globally odd ($\sigma_{kz} = -1$), but its dominant field components are spatially even with respect to the vertical midplane kz bisecting the waveguide channel.
15. K. Yamada, H. Morita, A. Shinya, and M. Notomi, Opt. Commun. **198,** 395 (2001).
16. M. Qiu, Phys. Rev. B **66,** 033103 (2002).
17. S. G. Johnson, M. L. Povinelli, P. Bienstman, M. Skorobogatiy, M. Soljacic, M. Ibanescu, E. Lidorikis, and J. D. Joannopoulos, in *Proceedings of 2003 Fifth International Conference on Transparent Optical Networks* (Institute of Electrical and Electronics Engineers, Piscataway, N.J., 2003), p. 103.