Quantum-Information Processing with Hybrid Spin-Photon Qubit Encoding (supplemental material)

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A. COOPER-PAIR BOXES

The scheme for quantum information processing described in the main text exploits a Cooper-pair box (CPB) [1, 2], whose Hamiltonian can be written in the basis of the charge states $|n\rangle$ (with $\hat{n}|n\rangle = n|n\rangle$) [3] as

$$\hat{H}_{CPB} = \sum_{n=-\infty}^{\infty} \left[4E_C (n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right) \right], \tag{1}$$

where E_J is the Josephson energy, E_C is the charging energy and n_g is the dimensionless gate charge. The numerical diagonalization of (1) with $n_g = 0.5$, $E_C = 4.9\ GHz$ and $E_J = 6.2\ E_C$ [4] yields a highly anharmonic spectrum. Hence, we can safely truncate the Hilbert space of the CPB to the lowest three levels shown in the central part of Fig. 2(b) of the main text. In this regime coherence times of the CPB are considerably longer than the CZ gating time [5] and the spectrum is sufficiently anharmonic to obtain high fidelities in our simulations.

B. PROOF-OF-PRINCIPLE EXPERIMENT

We describe here a simpler setup, which could be exploited for the first proof-of-principle experiments. This is formed by a CPB and two distinguishable spin $s = 1/2$ ensembles A and A' within the same cavity. We exploit the lowest three levels of the CPB: $|\psi_0^B\rangle$, $|\psi_1^B\rangle$ and $|\psi_2^B\rangle$, with transition energies ω^B and $\omega^{B'}$ ($\hbar = 1$ is assumed). The excitation energies of the spin ensembles are ω^A and $\omega^{A'}$. Two different harmonics of the resonator are taken into account, of frequency ω_c^A and $\omega_c^{A'}$. ω_c^A is intermediate between ω^A and ω^B, while $\omega_c^{A'}$ is intermediate between $\omega^{A'}$ and $\omega^{B'}$. In this way the cavity mode ω_c^A ($\omega_c^{A'}$) can be coupled both to the spin gap ω^A ($\omega^{A'}$) and to the superconducting gap ω^B ($\omega^{B'}$), by tuning its frequency. We recall here the definition of the logical 2-qubit states:

$$\begin{aligned}
|0_A 0_{A'}\rangle &\equiv |\psi_1^A, n_A = 0\rangle \otimes |\psi_1^{A'}, n_{A'} = 0\rangle \\
|0_A 1_{A'}\rangle &\equiv |\psi_1^A, n_A = 0\rangle \otimes |\psi_0^{A'}, n_{A'} = 1\rangle \\
|1_A 0_{A'}\rangle &\equiv |\psi_0^A, n_A = 1\rangle \otimes |\psi_1^{A'}, n_{A'} = 0\rangle \\
|1_A 1_{A'}\rangle &\equiv |\psi_0^A, n_A = 1\rangle \otimes |\psi_0^{A'}, n_{A'} = 1\rangle
\end{aligned} \tag{2}$$

We note that the CPB does not enter in the definition of the qubits: in the idle configuration it remains in its ground state $|\psi_0^B\rangle$.

Single-qubit rotations are implemented in a way similar to that described in the text, by means of off-resonant pulses and by tuning ω_c^γ to ω^γ for the proper amount of time ($\gamma = A, A'$).

By exploiting the hybrid encoding (2) the CZ gate can be implemented as in the scalable setup. Here, however, a single resonator is considered and thus photon-hopping does not occur. As stated in the text, CZ is performed with a two-step Rabi oscillation of the CPB between $|\psi_0^B\rangle$ and $|\psi_2^B\rangle$ accompanied by the absorption and emission of the

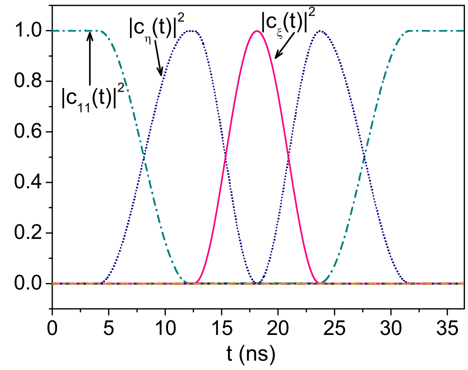

FIG. 1: (Color online) Calculated time-dependence of the components of $|\psi(t)\rangle$ during the simulation of a CZ in the simpler setup described for the first proof-of-principle experiments.

two photons entering the definition of the two qubits. This is done through the intermediate states:

$$|\eta\rangle \equiv |\psi_0^A, n_A = 0\rangle \otimes |\psi_1^B\rangle \otimes |\psi_0^{A'}, n_{A'} = 1\rangle$$
$$|\xi\rangle \equiv |\psi_0^A, n_A = 0\rangle \otimes |\psi_2^B\rangle \otimes |\psi_0^{A'}, n_{A'} = 0\rangle. \tag{3}$$

We now examine the effect of the three-step pulse sequence used to implement CZ on the four logical states (2).

- Let's start by considering the two qubits initially prepared in state $|1_A 1_{A'}\rangle$. First step: ω_c^A is tuned to ω^B by means of a π-pulse, which transfers the excitation from the photon A to the intermediate level $|\psi_1^B\rangle$ of the CPB, carrying the system to state $|\eta\rangle$. Second step: a 2π-pulse brings $\omega_c^{A'}$ into resonance with $\omega^{B'}$, thus inducing a full Rabi oscillation between states $|\eta\rangle$ and $|\xi\rangle$. Third step: the repetition of the first pulse brings back the system to $|1_A 1_{A'}\rangle$, with a phase π acquired during the Rabi flop in the second step. The extra phase due to the absorption/emission processes occurring in the first and third steps can be set to zero by properly choosing the delay between the two corresponding π-pulses.

- If the system is initially in $|1_A 0_{A'}\rangle$ there is only the photon of frequency ω_c^A and the 2π-pulse (second step) leaves the state $|\eta\rangle$ unaffected. The two π-pulses (first and third steps) brings the state from $|1_A 0_{A'}\rangle$ to $|\eta\rangle$ and back, but the phase acquired during this process is set to zero by the same delay fixed previously.

- If the system is in $|0_A 1_{A'}\rangle$, the two π-pulses do not induce any transition. In fact, in this state there is only a photon in mode A' which is never brought into resonance with the transition ω^B. Thus, also the Rabi swap (second step) is ineffective, as the CPB is always in its ground state.

- Finally, state $|0_A 0_{A'}\rangle$ is completely unaffected by the present sequence.

Hence, the sequence of pulses described above implements a CZ, i.e., in matrix representation on basis (2):

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

As photon-hopping is not required, the gating times are reduced with respect to the scalable setup, to about 30 ns (see the simulation reported in Fig. 1).

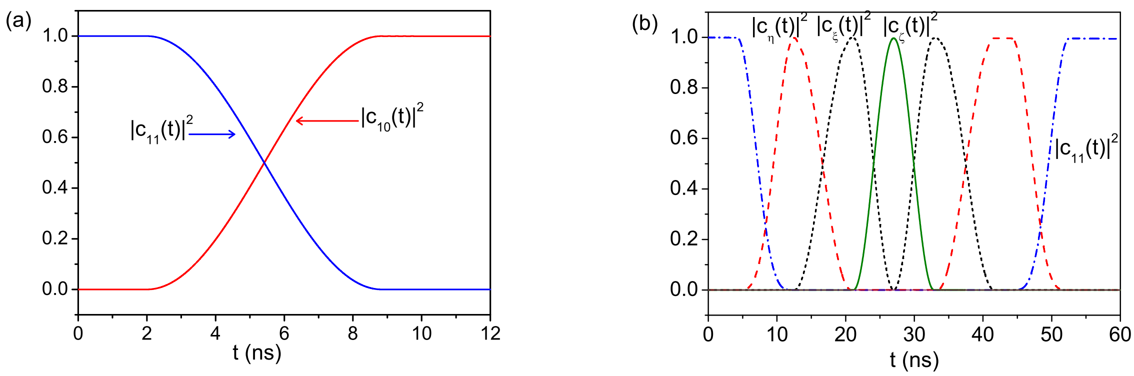

FIG. 2: (Color online) Calculated time-dependence of the main components of $|\psi(t)\rangle$ in a single-qubit rotation, (a), and in a CZ, (b), including a term of photon-leakage. Calculations are performed in Schrödinger picture.

C. PHOTON LOSS

Following [6], we include the photon leakage by modeling the evolution of the system as due to an effective Hamiltonian H_{eff} involving the complex cavity frequencies $\tilde{\omega}_c^\gamma = \omega_c^\gamma - i\Gamma$. With this substitution the photon part of the Hamiltonian changes into

$$\sum_\gamma \omega_c^\gamma(t)\hat{a}_\gamma^\dagger \hat{a}_\gamma \to \sum_\gamma \left[\omega_c^\gamma(t) - i\Gamma\right] \hat{a}_\gamma^\dagger \hat{a}_\gamma .$$

Here Γ is the cavity loss rate, assumed in the 10 kHz range [7], and ω_c^γ are the real frequencies introduced in the text. Fig. 2 shows the absolute squared value of the components of the system wave-function as a function of time, during the simulation of a $\hat{R}_x(\pi)$ and of a CZ gate. No significant difference can be noted with respect to Fig. 3(b) of the text: the non-unitary evolution due to H_{eff} leads only to a few photons lost over a thousand during our gating times. Finally, it has been experimentally shown that pure dephasing of the cavity modes is negligible (see, e.g., [8] where the measured value of the dephasing time T_2 approximately corresponds to twice the value of the photon decay time T_1).

D. PULSE SHAPE

In the simulations reported in the text the cavity modes are modulated by means of step-like pulses. We have checked that the same results can be achieved by linearly varying the resonator frequency on a timescale $\gg 1/\omega_c^\gamma(0)$: this regime is well within the validity of the rotating-wave approximation and no generation of unwanted photons due to dynamical-Casimir effect is observed [9].

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