Non-equilibrium delocalization-localization transition of photons in circuit QED

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We show that photons in two tunnel-coupled microwave resonators each containing a single superconducting qubit undergo a sharp non-equilibrium delocalization-localization (self-trapping) transition due to strong photon-qubit coupling. We find that dissipation favors the self-trapped regime and leads to the possibility of observing the transition as a function of time without tuning any parameter of the system. Furthermore, we find that self-trapping of photons in one of the resonators (spatial localization) forces the qubit in the opposite resonator to remain in its initial state (energetic localization). This allows for an easy experimental observation of the transition by local read-out of the qubit state.

In circuit quantum electrodynamics (QED), superconducting qubits are coupled with microwave photons in a transmission line resonator reaching extremely strong light-matter interactions within an integrated circuit [1]. Device integration, high tunability and individual addressability of each resonator make wide parameter regimes easily accessible. Circuit QED thus constitutes one of the most promising solid-state architectures for quantum information processing and offers the possibility to study fundamental questions of interacting quantum systems [2]. Initially, the focus has been on the control of the coupling between a single cavity and a single qubit and subsequent work demonstrated a great level of experimental control of single-cavity systems [3–7]. Now, a key challenge for scalability and further progress in the field is the understanding of small coupled systems, in particular effective qubit-qubit and photon-photon interactions and their interplay with dissipation [8–10].

In this letter, we study theoretically a photon Josephson junction (PJJ) consisting of two tunnel-coupled microwave resonators each containing a single superconducting qubit (Fig. 1). We show that photons undergo a sharp non-equilibrium delocalization-localization transition from a regime where an initial photon population imbalance between the two resonators undergoes coherent oscillations (delocalized) between the two resonators to another regime where it becomes self-trapped (localized) as the photon-qubit interaction is increased. Similar self-trapping transitions were found in optical fibres [12], molecules [13], cold atom [14–16] and polariton BEC's [17]. In all of these systems self-trapping is due to a Kerr/Bose-Hubbard like nonlinearity and has been experimentally observed in the semiclassical regime with a large number of particles. The circuit QED implementation proposed in this paper has several advantages with respect to these systems: (i) Self-trapping is due to a Jaynes-Cummings (JC) rather than a Kerr/Bose-Hubbard like nonlinearity. The JC interaction accurately describes the photon-qubit coupling in a microwave resonator [11]. In contrast, a Kerr/Bose-Hubbard like nonlinearity is often a rather crude approximation of the experimental conditions [18, 19]. (ii) The PJJ is a genuinely dissipative system. We show that dissipation favors the localized regime and leads to the possibility of observing the localization transition experimentally as a function of time without tuning any parameter of the Hamiltonian. (iii) The PJJ may operate in the semiclassical (many photons) as well as quantum (few photons) limit since each resonator can initially be pumped with an almost arbitrary number of photons.

In the following, we study in detail the classical versus quantum nature of this transition and present numerical as well as analytical results including the effects of dissipation. At the end of the paper, we outline a precise proposal on how to measure the localization transition of photons experimentally.

We describe the PJJ (Fig. 1) by a two-site Jaynes-Cummings-Hubbard Hamiltonian (JCHM) or Jaynes-Cummings dimer (JCD)

$$H = \sum_{i=\mathrm{L,R}} h_i^{\mathrm{JC}} - J(a_{\mathrm{L}}^\dagger a_{\mathrm{R}} + \mathrm{h.c.})\,, \tag{1}$$

where h_i^{JC} denotes the local JC Hamiltonian $h_i^{\mathrm{JC}} = \omega_c\, a_i^\dagger a_i + \omega_x \sigma_i^+ \sigma_i^- + g(\sigma_i^+ a_i + \sigma_i^- a_i^\dagger)$ for the left (L) or the right (R) cavity, a_i $(a_i^\dagger)$ and σ_i^+ (σ_i^-) are the photon creation (annihilation) and qubit raising (lowering) operators, respectively. The photon mode frequency is ω_c, the qubit transition energy is ω_x and the photon-qubit coupling is given by g (we set $\hbar = 1$). The photon-qubit interaction induces an anharmonicity in the spectrum of the JC Hamiltonian which leads to an effective

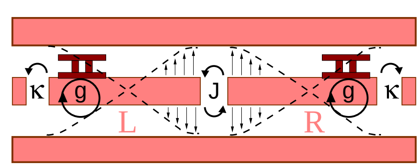

FIG. 1: Schematic diagram of the photonic Josephson junction (PJJ) proposed in this paper. Two transmission line microwave resonators (L,R) are coupled in series with a tunneling rate J, determined by the series capacitance of the resonators. Each resonator is strongly coupled to a superconducting qubit with a coupling rate g, providing a strong JC non-linearity. Photons can leave each resonator at a rate κ, providing a mechanism for dissipation.

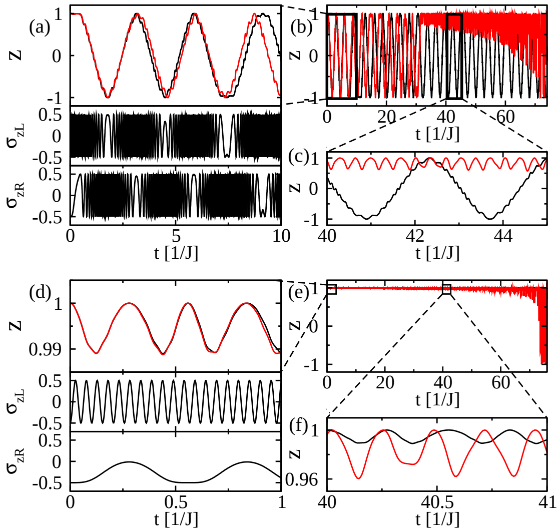

FIG. 2: Photon imbalance $z(t)$ and inversion of the qubits $\sigma_{z(\mathrm{L,R})}(t)$ obtained from a semiclassical approximation for the dissipative JCD with $\kappa = 0$ (black curves) and $\kappa = 0.05J$ (red curves) for initially $n_\mathrm{L}(0) = N(0) = 20$ photons (and qubits initially in their groundstate $\sigma_{z(\mathrm{L,R})}(0) = -1/2$). Shown are results in the Josephson regime with $g = 0.9g_c$ (Figs. (a),(b),(c)) and in the self-trapped regime with $g = 1.1g_c$ (Figs. (d),(e),(f)). Figures (a),(c) and (d),(f) are zoom-in's corresponding to the boxes in figure (b) and (e). Here, the photon number scales as $N(t) = N(0)e^{-\kappa t}$.

on-site repulsion (anti-bunching) for photons. Throughout the paper we will assume zero detuning ($\omega_x = \omega_c$) for which this anharmonicity is strongest. The JCHM has originally been introduced to describe a superfluid-Mott transition of polaritons in an infinite array of cavities [21–28]. Recently, dynamical aspects have been investigated restricted to the one excitation (either photon or qubit) subspace and neglecting dissipation [29–31]. In the JCD discussed here, cavity photon dissipation is taken into account by a Lindblad master equation for the system's density matrix ρ

$$\frac{\partial \rho}{\partial t} = i[\rho, H] + \frac{\kappa}{2} \sum_{i=\mathrm{L,R}} \left(a_i \rho a_i^\dagger - a_i^\dagger a_i \rho - \rho a_i^\dagger a_i \right) , \quad (2)$$

where κ^{-1} is the photon life time in each cavity. Using a Fock state basis the master equation is solved for up to 20 photons. The central quantity of interest is the population imbalance $z(t) = (n_\mathrm{L}(t) - n_\mathrm{R}(t))/N(t)$ with $n_i = \mathrm{Tr}\, \hat{a}_i^\dagger \hat{a}_i \hat{\rho}$ and the total photon number $N = n_\mathrm{L} + n_\mathrm{R}$. Throughout the paper, we consider the experimentally most relevant initial condition where the left cavity is pumped with N photons, the right cavity is empty and both qubits reside in their respective groundstate at $t = 0$. For large photon numbers, we can resort to a semiclassical approximation in which correlation functions in Eq. (2) are decoupled by simple factorization, e.g., $\langle a^\dagger \sigma^- \rangle \approx \langle a^\dagger \rangle \langle \sigma^- \rangle$, yielding eight coupled equations of motion for the expectation values of the photon and qubit operators which can be solved for an arbitrary number of photons. In the following, we present results of the semiclassical approximation (Fig. 2) as well as the full quantum solution of Eq. (2) (Fig. 3). In both cases, we have chosen the same number of initial photons for better comparison [33].

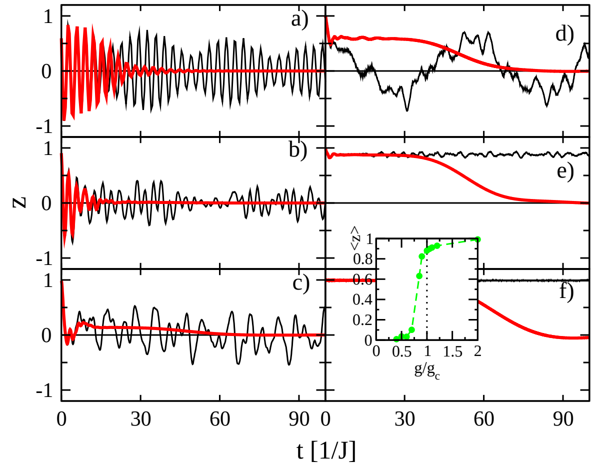

FIG. 3: Photon imbalance $z(t)$ obtained from a full numerical solution of the master equation (2) (including quantum fluctuations) with $\kappa = 0$ (black curves) and $\kappa = 0.05J$ (red curves) for initially $N(0) = 20$ photons (and qubits initially in their groundstate $\sigma_{z(\mathrm{L,R})}(0) = -1/2$). Figs. (a)-(f) show results for $g = 0.1, 0.4, 0.6, 0.8, 0.9, 2g_c$. The inset shows the time-averaged imbalance $\langle z \rangle$ (averaged over the time interval $t \in [0, 100/J]$) without dissipation ($\kappa = 0$) (dashed line) as a function of the photon-qubit coupling g normalized with the semiclassical critical value g_c in Eq. 4. In comparison, the semiclassical transition at this scale is essentially abrupt (dotted line).

We first discuss results of the semiclassical approximation without dissipation ($\kappa = 0$). In this case one can further reduce the number of coupled equations using energy conservation and the specific initial condition chosen above, yielding only four equations of motions

$$\begin{aligned} \dot{\theta}_\mathrm{L} &= -2g\mathrm{Re}(\psi_\mathrm{L}) \\ \dot{\theta}_\mathrm{R} &= -2g\mathrm{Im}(\psi_\mathrm{R}) \\ \mathrm{Re}(\dot{\psi}_\mathrm{L}) &= g\sin(\theta_\mathrm{L})/2 - J\mathrm{Im}(\psi_\mathrm{R}) \\ \mathrm{Im}(\dot{\psi}_\mathrm{R}) &= g\sin(\theta_\mathrm{R})/2 + J\mathrm{Re}(\psi_\mathrm{L}) , \end{aligned} \quad (3)$$

where $\psi_{(\mathrm{L,R})} = \langle a_{(\mathrm{L,R})} \rangle$, $\mathrm{Im}(\psi_\mathrm{L}) = \mathrm{Re}(\psi_\mathrm{R}) = 0$ and the angles θ_i describe the qubit states $\langle \vec{\sigma}_\mathrm{L} \rangle = -(\sin\theta_\mathrm{L}, 0, \cos\theta_\mathrm{L})/2$ and $\langle \vec{\sigma}_\mathrm{R} \rangle = -(0, \sin\theta_\mathrm{R}, \cos\theta_\mathrm{R})/2$. At zero interaction ($g = 0$) Eqs. (3) are exactly solvable. In this limit the photon imbalance undergoes coherent harmonic oscillations $z(t) = \cos \omega_J t$ with frequency $\omega_J = 2J$. As g becomes nonzero this frequency decreases and the oscillations become anharmonic. At a critical value of the coupling constant

$$g_c \approx 2.8\sqrt{N}J \quad (4)$$

the period of oscillation diverges (critical slowing down) and an abrupt transition occurs to a localized regime, where the initial photon imbalance stays almost completely in the left cavity, i.e., $z(t) \approx 1$. Solutions of Eq. (3) near the transition ($g \sim g_c$) are shown in Fig. 2. An important and useful result is that the localization transition of photons can effectively be observed in the population inversions of the two qubits, which depend on the number of photons in each cavity. In the delocalized regime ($g < g_c$), when photons are tunneling, e.g., from the left into the right cavity, Rabi oscillations of the left qubit slow down considerably while those of the right qubit speed up (Fig. 2(a)). After half a tunneling period the scenario is reversed. On the other hand, in the localized regime ($g > g_c$) the left qubit displays fast, complete Rabi oscillations while the right one displays slow, small amplitude oscillations (Fig. 2(d)). In other words, spatial localization of photons in the left resonator induces an energy state localization of the right qubit (notice that deep in the localized regime ($g \gg g_c$) the right qubit remains very close to the ground-state at all times, i.e., $\sigma_{z(\mathrm{L,R})}(t) \approx \sigma_{z(\mathrm{L,R})}(0)$). This suggests that the localization transition of photons in a PJJ can be observed experimentally by a local readout of the qubit states. It also shows that qubit-qubit correlations are largely suppressed in the localized regime.

Taking into account dissipation within the semiclassical approximation (i.e., solving all eight coupled equations of motion) leads to two important effects : (i) Dissipation stabilizes the localized regime, i.e., we find self-trapped solutions for significantly smaller values of the photon-qubit coupling $g < g_c$; (ii) Dissipation also leads to the possibility of observing the localization transition dynamically, i.e., in time without tuning any parameter of the Hamiltonian. This is shown in Fig. 2(b). If the interaction parameter is slightly below the critical value ($g = 0.9g_c$), the system first undergoes coherent, large amplitude oscillations but switches to self-trapping after a critical time $t_{c_1} \approx 30/J$. When less than two photons are left in the PJJ (zero effective photon on-site repulsion) at $t_{c_2} \approx 70/J$, the qubit Rabi oscillations induce single-photon Rabi oscillations between the cavities with amplitude $z(t) \approx 1$. If, on the other hand, the interaction strength is above the critical value ($g = 1.1g_c$), the system always remains self-trapped as long as $N(t) > 1$ (before it enters the regime of single-photon Rabi oscillations at t_{c_2}). This interesting behaviour can be explained with the different scaling of the qubit-photon interaction ($\sim g\sqrt{N}$) and the tunneling term ($\sim JN$) with photon number N, which leads to the square-root dependence of the critical interaction strength $g_c \sim \sqrt{N}J$ (see Eq. (4)). Consequently, as the system starts to dissipate photons, g_c effectively decreases and the system can switch to self-trapping at a certain time $t_{c_1} > 0$, irrespective of the fact that we initially started in the delocalized regime with $g < g_c$. Notice that this effect is fundamentally different from a Kerr/Bose-Hubbard like nonlinearity (with Hubbard parameter U), where the repulsive interaction scales like $\sim UN^2$ and thus the critical interaction strength is inversely proportional to the particle number ($U_c \sim J/N$).

Full numerical solutions (including quantum fluctuations) of the master equation (2) are shown in Fig. 3. Most importantly, we observe that the localization transition survives. However, specific quantum correction show up: (i) Deep inside the delocalized regime (see Fig. 3(a)) the photon imbalance displays beatings of the coherent oscillations, a quan-

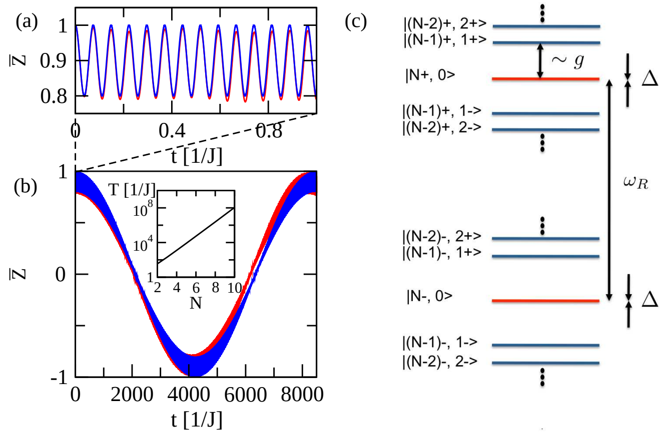

FIG. 4: Rescaled photon imbalance $\tilde{z} = (n_1(t) - n_2(t))/N(0)$ deep in the self-trapped regime for $N(0) = 5$ photons with $g = 3g_c$ and $\kappa = 0$. Shown are small amplitude Rabi oscillations on short time scales (Fig. 4(a)) and large amplitude ultra-long tunneling (Fig. 4(b)). Results of a full numerical solution of the quantum master equation (2) (red curve in Figs. 4(a) and 4(b)) are compared with strong-coupling degenerate perturbation theory (sdPT) (blue curve) based on the effective level scheme shown in in Fig. 4(c). Here, $|M\sigma, K\mu\rangle = \{|M\sigma\rangle_L|K\mu\rangle_R, |K\mu\rangle_L|M\sigma\rangle_R\}$ denotes the pair of degenerate polariton eigenstates of the Hamiltonian (1) at $J = 0$ with (M, K) lower/upper $(\sigma, \mu = \pm)$ polaritons in left (L) and right (R) cavity, respectively. Note that a polariton eigenstate $|M\sigma\rangle$ is a mixed photon $(M, M-1)$ - qubit (g, e) state, i.e, $|M\sigma\rangle = (|M\,, g\rangle + \sigma|(M-1)\,, e\rangle)/\sqrt{2}$ (for zero detuning ($\omega_x = \omega_c$) and $M > 0$; the zero polariton state is a special case with $|0\rangle \equiv |0-\rangle = |0\,, g\rangle$). Their degeneracy is lifted due to tunneling J, which induces the splitting Δ. The short and long time dynamics deep inside the localized regime (Figs. 4(a) and 4(b)) can be explained quantitatively using this effective level scheme together with sdPT. The frequency of the Rabi oscillations (due to exchange of one photon between local qubit and cavity) is given by the large splitting between lower and upper polariton states which yields for large photon numbers $\omega_R = 2g\sqrt{N} + \mathcal{O}(1/N)$. The period of ultra-long tunneling is set by the splitting Δ, which we have calculated in leading order from N-th order sdPT yielding $\Delta = c_N J(J/g)^{N-1}$ with a constant c_N that depends on the number of photons N. The inset in Fig. 4(b) shows the scaling of the corresponding time period $T = 2\pi/\Delta$ of the large amplitude oscillations as a function of N. Neglecting the perturbative corrections to the eigenvectors of the Hamiltonian (1), we find for the rescaled imbalance $\tilde{z} = \cos(\Delta t)\left[1 - (1/N)\sin^2(\omega_R t)\right] + \mathcal{O}(J^2/g^2)$ which quantitatively reproduces the numerical results in Figs. 4(a) and 4(b) (red curve).

tum feature that is absent in semiclassical solutions. Dissipation strongly damps the large amplitude oscillations and suppresses these beats if the dissipation time is smaller than the beating time; (ii) the localization transition is shifted to smaller g values and smoothened (see inset Fig. 3); (iii) in the localized regime with dissipation (see Figs. 3(e,f)) the imbalance approaches zero asymptotically at long times displaying no single-particle Rabi oscillations. (iv) due to the strong damping of the initial large amplitude oscillations, the clear switching behavior observable in the semiclassical limit for $g < g_c$ (Fig. 2(b)) is washed out while below the transition (see Fig. 3(c)), the system still reaches a small non-zero average of the imbalance at intermediate times. (v) the deeply localized regime displays rich multi-scale time dynamics, which can be explained using the effective level scheme shown in Fig. 4(c) (for a detailed explanation see caption in Fig. 4). For short times, Rabi oscillations of the qubit at a frequency $\omega_R \approx 2g\sqrt{N}$ (corresponding to the large splitting between upper and lower polariton states) induce small amplitude oscillations (on the order $\sim 1/N$) of the rescaled photon imbalance $\bar{z}(t)$ (see caption in Fig. 4) at the same frequency. However, at ultra-long times the localization of photons is unstable and almost complete oscillations of the imbalance set in. The small frequency of these oscillations is given by the splitting $\Delta \propto J(J/g)^{N-1}$ of two degenerate polariton states due to tunneling. The period $T = 2\pi/\Delta$ of these oscillations is found to increase very fast (quasi-exponentially) with increasing photon number (see inset in Fig. 4(b)), Thus, true localization disappears in a small system, but turns exponentially 'good' with increasing system size (photon number). Already for five photons the ultra-long tunneling regime is hardly accessible experimentally. Ultra-long tunneling times were also predicted for the BHM [32]. We should note, however, that under experimental conditions any asymmetry between the two cavities (e.g., due to small differences in detunings or coupling constants) will lift this degeneracy and set the effective time period of the large amplitude oscillations. A corollary of this statement is that as long as such an asymmetry is much smaller than the frequency $\omega_J = 2J$ the localization transition should be observable.

The physics of a PJJ should be readily observable using a circuit QED implementation with realistic device parameters. A possible device consists of two series-coupled transmission line resonantors, each containing a single superconducting qubit (Fig. 1). A broad parameter space is available through changes in lithographic patterning. In particular, qubit-cavity coupling g can range from 1 MHz to 300 MHz, while cavity-cavity coupling J and cavity dissipation rate κ can be tuned independently in a range 50 kHz to 50 MHz. An experimental observation of the localization transition proceeds in three parts. One cavity is populated with an initial photon population (initialization), evolution proceeds for a fixed duration of time (evolution), and the photon occupancy of each cavity is finally measured (read-out). This entire process would be repeated for varying evolution times, thus allowing full reconstruction of the population imbalance $z(t)$. Initialization can be accomplished using three different methods: (i) In the simplest method, the cavity is populated with a coherent photon state while the qubit is far off resonance ($\omega_x \ll \omega_c$); the qubit is then quickly brought into resonance ($\omega_x = \omega_c$) for the evolution [4, 5]. This scenario is best described by the results of the semiclassical approximation in Fig. 2. (ii) The cavity-qubit system can also be populated directly with a polariton state, i.e., an eigenstate of the JC Hamiltonian, using a properly timed π-pulse [6]. (iii) Finally, a N-photon Fock state can be constructed sequentially by successively exciting the qubit very quickly and bringing it into resonance ($\omega_x = \omega_c$). The multi-photon/polariton transitions in the latter two scenarios are resolvable up to $5-10$ excitations and are faithfully realized by the initial conditions chosen in Fig. 3 (full quantum calculation). After evolution for a given time, the qubits coupled to each cavity will be used to measure the photon occupation of that cavity. When strongly coupled, the qubit frequency is shifted depending on the number of photons in the cavity [3]; by interrogating these different frequencies, a quantum non-demolition experiment of photon number can be performed [7].

In this work we have shown that two tunnel-coupled microwave resonators each containing a Jaynes-Cummings type nonlinearity undergo simultaneously sharp localization transitions of photons (spatial) and qubits (energetic). We have shown that dissipation can drive this transition if the coupling constants are properly chosen without the necessity of tuning any parameter of the system. Our results suggest many directions of further theoretical investigations including effects of detuning, quantum-classical crossover and interplay of localization and entanglement.

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[1] A. Wallraff *et al.*, Nature **431**, 162 (2004).
[2] R. J. Schoelkopf and S. M. Girvin, Nature **451**, 664 (2008).
[3] D. I. Schuster *et al*, Nature 445, 515-518 (2007).
[4] M. A. Sillanpaa *et al*, Nature 449, 438-442 (2007).
[5] M. Hofheinz *et al*, Nature 454, 310-314 (2008).
[6] L. S. Bishop *et al*, Nat. Phys. 5, 105-109 (2009).
[7] B. R. Johnson *et al*, arXiv:1003.2734 (2010).
[8] J. Majer *et al.*, Nature **449**, 443 (2007).
[9] J. M. Fink *et al*, Phys. Rev. Lett. **103**, 083601 (2009).
[10] D. Gerace *et al*, Nature Phys. **5**, 281 (2009).
[11] J. M. Fink *et al*, Nature **454**, 315 (2008).
[12] S. M. Jensen, IEEE J. Quantum Electron. QE-18, 1580 (1981)
[13] J.C. Eilbeck, P.S. Lomdahl and A.C. Scott, Physica D **16**, 318 (1985).
[14] A. Smerzi *et al.*, Phys. Rev. Lett. **79**, 4950 (1997).
[15] M. Albiez *et al.*, Phys. Rev. Lett. **95**, 010402 (2005).
[16] S. Levy *et al.*, Nature Lett. **449**, 579 (2007).
[17] D. Sarchi *et al*, Phys. Rev. B **77**, 125324 (2008).
[18] I. Zapata, F. Sols, and A. J. Leggett, Phys. Rev. A **67**, 021603(R) (2003).
[19] M. T. Martinez, A. Posazhennikova, and J. Kroha, Phys. Rev. Lett. **103**, 105302 (2009).
[20] J. M. Fink *et al*, Nature 454, 315-318 (2008).
[21] A. D. Greentree *et al.*, Nature Phys. **466**, 856 (2006).
[22] M. J. Hartmann, F. G. S. L. Brandao, and M. B. Plenio, Nature Phys. **462**, 849 (2006).
[23] D. G. Angelakis, M. F. Santos, and S. Bose, Phys. Rev. A **76**, 031805(R) (2007).
[24] D. Rossini and R. Fazio, Phys. Rev. Lett. **99**, 186401-1 (2007).
[25] M. Aichhorn, M. Hohenadler, C. Tahan, and P. B. Littlewood, Phys. Rev. Lett. **100**, 216401 (2008).
[26] S. Schmidt and G. Blatter, Phys. Rev. Lett. **103**, 086403 (2009).
[27] J. Koch and K. Le Hur, Phys. Rev. A **80**, 023811 (2009).

[28] S. Schmidt and G. Blatter, Phys. Rev. Lett. **104**, 216402 (2010).
[29] C. D. Ogden, E. K. Irish, and M. S. Kim, Phys. Rev. A **78**, 063805 (2008).
[30] M. I. Makin *et al*, Phys. Rev. A **80**, 043842 (2009).
[31] Ke Zhang and Zhi-Yuan Li, Phys. Rev. A **81**, 033843 (2010).
[32] L. Bernstein, J. Eilbeck, A. Scott, Nonlinearity 3, 293. (1990).
[33] Rescaling of the time parameter $\tilde{t} = t/N$ and the coupling constants $\tilde{g} = g\sqrt{N}$, $\tilde{J} = JN$ and $\tilde{\kappa} = \kappa N$ renders the semi-classical equations of motion independent of the initial photon number N since $\psi_L(0) = \sqrt{N}$ and $\psi_R(0) = 0$.