A general law for electromagnetic induction

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Abstract – The definition of the induced *emf* as the integral over a closed loop of the Lorentz force acting on a unit positive charge leads immediately to a general law for electromagnetic induction phenomena. The general law is applied to three significant cases: moving bar, Faraday's and Corbino's disc. This last application illustrates the contribution of the drift velocity of the charges to the induced *emf*: the magneto-resistance effect is obtained without using microscopic models of electrical conduction. Maxwell wrote down "general equations of electromotive intensity" that, integrated over a closed loop, yield the general law for electromagnetic induction, if the velocity appearing in them is correctly interpreted. The flux of the magnetic field through an arbitrary surface that have the circuit as contour *is not the cause* of the induced *emf*. The flux rule must be considered as a calculation shortcut for predicting the value of the induced *emf* when the circuit is filiform. Finally, the general law of electromagnetic induction yields the induced *emf* in both reference frames of a system composed by a magnet and a circuit in relative uniform motion, as required by special relativity.

Introduction. – Electromagnetic induction phenomena are generally described by the "flux rule", usually referred to as the Faraday-Neumann law[1]:

$$\mathcal{E} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{B} \cdot \hat{n}\, \mathrm{d}S \qquad (1)$$

($\mathcal{E}$ induced *emf*; $\vec{B}$ magnetic field; S any surface that has the closed loop of the electrical circuit as contour). However, it is sometimes acknowledged that the flux rule presents some problems when part of the electrical circuit is moving. Some authors speak of exceptions to the flux rule [2]; others save the flux rule by *ad hoc* choices of the integration path over which the induced *emf* is calculated [3]. The validity of the flux rule has been advocated also in recent papers [4,5]: in both cases the flux rule is *assumed* to be valid and the authors manage to show how it works in several critical situations. Finally, it is to be stressed that, in the literature, the possible contribution to the induced *emf* of the drift velocity of the charges is completely ignored. As shown in this paper, this is correct only when the electrical circuit is filiform (or equivalent to a filiform circuit; see below the case of a bar moving in a magnetic field): when part of the circuit is made of extended conductor, the drift velocity yields a contribution (see, below, the treatments of Corbino and Faraday disc).

The approach taken in the present paper is radically different and based on the definition of the induced *emf* given in eq. (3): it leads immediately to a "general law" for electromagnetic induction phenomena that is applied, for illustration, to three significant cases (moving bar, Faraday and Corbino disc). Then, it is shown that the flux rule is neither a field law nor a causal law: it must be considered as a calculation shortcut when the electrical circuit is filiform (or equivalent to). Finally, it is recalled that Maxwell wrote down "general equations of electromotive intensity" that, integrated over a closed loop, yield the "general law" for electromagnetic induction derived in this paper, if the velocity appearing in Maxwell equations is correctly interpreted.

The matter has basic conceptual relevance, not confined to physics teaching; it has also historical and epistemological aspects that deserve to be discussed[2].

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[1]It is worth stressing that the theory of electromagnetic induction developed by Faraday in his *Experimental Researches* is a *field* theory, while the flux rule is not (see below). Faraday states that there is induced current when there is relative motion between conductor and "lines of magnetic force" conceived as real physical entities [1].

[2]The treatment of induction phenomena expounded in this paper has been firstly presented in a communication to the XXXIX Congress of the AIF (Associazione per l'Insegnamento della Fisica —Association for the Teaching of Physics) [6]; then, in a lecture during an in service training of high-school teachers [7]; it appears

The "law" of electromagnetic induction. – Let us begin with the acknowledgement that the expression of Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad (2)$$

not only gives meaning to the fields solutions of Maxwell equations when applied to point charges, but yields new predictions.

The velocity appearing in the expression of Lorentz force is the velocity of the charge: from now on, we shall use the symbol $\vec{v}_c$ for distinguishing the charge velocity from the velocity $\vec{v}_l$ of the circuit element that contains the charge.

Let us consider the integral of $(\vec{E} + \vec{v}_c \times \vec{B})$ over a closed loop

$$\mathcal{E} = \oint_l (\vec{E} + \vec{v}_c \times \vec{B}) \cdot \mathrm{d}l. \qquad (3)$$

This integral yields, numerically, the work done by the electromagnetic field on a unit positive point charge along the closed path considered. It presents itself as the *natural* definition of the electromotive force, within the Maxwell-Lorentz theory: $emf = \mathcal{E}$.

Since

$$\vec{E} = -\operatorname{grad}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad (4)$$

(φ scalar potential; $\vec{A}$ vector potential) we get immediately from eq. (3)

$$\mathcal{E} = -\oint_l \frac{\partial \vec{A}}{\partial t} \cdot \mathrm{d}l + \oint_l (\vec{v}_c \times \vec{B}) \cdot \mathrm{d}l. \qquad (5)$$

This is the "general law" for electromagnetic induction: its two terms represent, respectively, the contribution to the induced emf of the time variation of the vector potential and the effect of the magnetic field on moving charges. If we write $\vec{v}_c = \vec{v}_l + \vec{v}_d$, where $\vec{v}_l$ is the velocity of the circuit element and $\vec{v}_d$ the drift velocity of the charges[3], eq. (5) becomes

$$\mathcal{E} = -\oint_l \frac{\partial \vec{A}}{\partial t} \cdot \mathrm{d}l + \oint_l (\vec{v}_l \times \vec{B}) \cdot \mathrm{d}l + \oint_l (\vec{v}_d \times \vec{B}) \cdot \mathrm{d}l. \qquad (6)$$

Equation (6) shows that the drift velocity gives, in general, a contribution to the induced emf: if the circuit is filiform, the drift velocity contribution is null since $\vec{v}_d$ is parallel to $\mathrm{d}l$ (and, therefore, $(\vec{v}_d \times \vec{B}) \cdot \mathrm{d}l = 0$); however, when a part of the circuit is made by an extended material, the contribution of the drift velocity must be taken into account (see below for discussion of particular cases).

Equation (6) can be written in terms of the magnetic field[4]:

$$\mathcal{E} = \left[-\frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{B} \cdot \hat{n}\,\mathrm{d}S - \oint_l (\vec{v}_l \times \vec{B}) \cdot \mathrm{d}l \right]$$
$$+ \left\{ \oint_l (\vec{v}_l \times \vec{B}) \cdot \mathrm{d}l + \oint_l (\vec{v}_d \times \vec{B}) \cdot \mathrm{d}l \right\}. \qquad (7)$$

We have grouped under square and curly brackets the terms arising from the first and second term of equation (5), respectively. This grouping is fundamental for the physical interpretation of eq. (7). The interpretation reads

1) When the magnetic field does not depend on time, the sum of the two terms under square brackets is null, because is null the first term of eq. (5) from which they derive. In this case, the only source of the induced emf is the motion of the charges in the magnetic field.

2) If one overlooks this fundamental physical point and, consequently, reads eq. (7) as

$$\mathcal{E} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{B} \cdot \hat{n}\,\mathrm{d}S$$

in the case of a filiform circuit (for which the contribution of the drift velocity is null), one gets again the flux rule. This illustrates why the flux rule is predictive in these cases, notwithstanding the basic fact that it completely obscures the physical origin of the induced emf.

The flux rule: neither a field law nor a causal law. – The flux rule is not a field law. As a matter of fact, it connects what is happening at the instant t on a surface that have the circuit (closed integration path) as contour to what is happening, at the same instant, in the circuit: this implies an action at a distance with infinite velocity. It is not a causal law, because it connects what is happening in the circuit to what is happening on an *arbitrary* surface that has the circuit as a contour.

Furthermore, we have seen that the flux rule, also when correctly predictive, obscures the physical origin of the induced emf. For these reasons, the flux rule must be considered only as a calculation shortcut.

Moving bars and rotating discs. – As significant cases of application of the general law we shall consider the "moving bar" (fig. 1) and the "Faraday disc" (fig. 2). In the case of the moving bar, the general law (6) says that an emf equal to vBa is induced. This result comes out from the second integral containing the velocity $\vec{v}_l$

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also in an Italian textbook [8] and, sketchily, in [9]. All these reports are in Italian. It may be worthwhile to present this treatment in an international Magazine.

[3]We can use here the Galilean composition of velocities because $v_l \ll c$ and $v_d \ll c$.

[4]This transformation uses the relation $\vec{B} = \nabla \times \vec{A}$, the Stokes theorem and takes into account the fact that the circuit element $\mathrm{d}l$ moves with velocity $\vec{v}_l$: this last condition is responsible for the term $(-\oint_l (\vec{v}_l \times \vec{B}) \cdot \mathrm{d}l)$ under square brackets. See, for instance, [5] or [10].
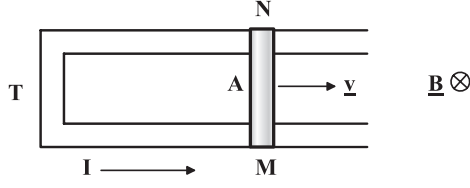
Fig. 1: The metallic bar A, a long, is sliding on a U shaped metallic frame T at constant velocity $\vec{v}$ in a constant and uniform magnetic field B perpendicular to the plane of the figure and entering the page.
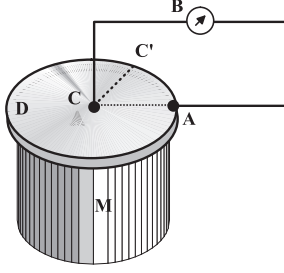

Fig. 2: Faraday disc. M is a cylindrical magnet; D a conducting disc (electrically isolated from the magnet). The external circuit has sliding contacts on the disc in A and C.

of the circuit element; the first integral is null since the magnetic field is constant; also the third integral is null, since, owing to the Hall effect, the drift velocity of the charges is always directed along the circuit element $\vec{\mathrm{d}l}$. The general law says also that the physical origin of the induced emf is the motion of the bar in the magnetic field and that the induced emf is localized into the bar. In spite of widespread beliefs[5], the localization of the induced emf is a significant physical matter. The emf is localized in that part of the circuit in which the current enters from the point at lower potential (point M in the case of the bar) and exits from the point at higher potential (point N in the case of the bar). This fact allows to treat the circuit of fig. 1 as a (quasi)steady current circuit in which the bar acts as a battery.

Let us now recall how the flux rule deals with this case. It predicts an emf given by vBa. In the light of the general law (7) and of its discussion, we understand why the flux rule predicts correctly the value of the emf: the reason lies in the fact that two line integrals (one under square and the other under curly brackets) cancel, *algebrically*, each other. However, we have shown above that the physics embedded in eq. (7) forbids to read equation $\mathcal{E} = vBa$ as the result of

$$\mathcal{E} = vBa + (-vBa + vBa) + 0 = vBa, \qquad (8)$$

that leaves operative the first term coming from the flux variation. Finally, on the basis of the flux rule, we are not

Table 1: Phenomena observed by Faraday with the disc; see fig. 2. The reference frame is the laboratory.

What is moving?	Relative motion disc-magnet	Induced current
Disc	Yes	Yes
Magnet	Yes	No
Disc and magnet	No	Yes

able to predict where the induced emf is localized: we can only guess that it is localized into the bar, since the bar is moving; but we are not able to prove it.

The case of Faraday disc is more complicated. First of all, we have, in this case, a part of the circuit (the disc) made of extended material: therefore, we expect a contribution to the induced emf from the drift velocity of the charges. We shall ignore here this contribution: we shall deal with it below.

Faraday carried out three qualitative experiments, summarized in table 1 [12,13][6]. Applying the general law (6) to the fixed integration path $ABCA$ or $ABCC'A$ (and ignoring the contribution from the drift velocity), we easily find the value of the radial induced emf (along any radius; the circuit element $C'A$ gives a null contribution)[7]: $\mathcal{E} = (1/2)B\omega R^2$, where B is the magnetic field (supposed uniform), R the disc radius and ω the angular velocity of the disc; when the disc is still, the induced emf is null. For applying the flux rule, we must choose the integration path $ABCC'A$ and consider the radius CC' as being in motion in order to have an increasing area given by $(1/2)(\omega t)R$ through whom calculate the magnetic flux (integration path chosen *ad hoc*). As in the case of the moving bar, the physics embedded in eq. (7) forbids an interpretation of the mathematical result in terms of flux variation: again the physical origin of the induced emf is due to the intermediacy of the magnetic component of Lorentz force.

The "prediction" of well-known experimental facts: Corbino's disc. – The following discussion will show how the charge drift velocity plays its role in the building up of the induced emf. In 1911, Corbino studied theoretically and experimentally the case of a conducting disc with a hole at its center (fig. 3) [14,15]. The first theoretical treatment of this case is due to Boltzmann who wrote down the equations of motion of charges in combined electric and magnetic fields [16]. Corbino, apparently not aware of this fact, obtained the same equations already developed by Boltzmann. However, while Boltzmann focused on magneto-resistance effects,

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[5]Einstein too, in his paper on special relativity states that "Moreover, questions as to the 'seat' of electrodynamic electromotive forces (unipolar machines) now have no point" [11].

[6]Faraday states that, when the magnet rotates, the "lines of magnetic force" stand still; the "lines" moves with the magnet only in translational motion.

[7]The first integral of equation (6) is null since the magnetic field is constant; the velocity appearing in the second integral is the velocity of the charges ωr due to the motion of the disc; the contribution of the third integral is ignored (it will be taken into account below).
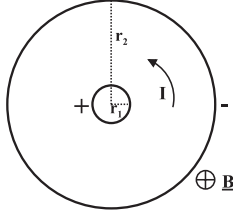
Fig. 3: Corbino disc. A conducting disc of radius r_2 has a circular hole at its center of radius r_1; highly conducting electrodes cover the inner and outer circular periphery. A battery connected to the inner and outer periphery, produces a radial current in the disc. When a static magnetic field $\vec{B}$ is applied perpendicularly to the disc and entering the page, a circular current arises in the direction shown by the arrow.

Corbino interpreted the theoretical results in terms of radial and circular currents and studied experimentally the magnetic effects due to the latter ones[8,9].

The application of the general law of electromagnetic induction to this case leads to the same results usually obtained (as Boltzmann and Corbino did) by writing down and solving the equations of motion of the charges in an electromagnetic field (by taking into account, explicitly or implicitly, the scattering processes).

If I_{radial} is the radial current, the radial current density $J(r)$ will be

$$J(r) = \frac{I_{radial}}{2\pi r s} \qquad (9)$$

and the radial drift velocity

$$v(r)_{drift} = \frac{I_{radial}}{2\pi r s n e}, \qquad (10)$$

where s is the thickness of the disc, n the electron concentration and e the electron charge. According to the general law (6), the induced emf around a circle of radius r is given by

$$\mathcal{E}_{circular} = \oint_0^{2\pi r} (\vec{v}(r)_{drift} \times \vec{B}) \cdot \vec{dl} = \frac{I_{radial} B}{s n e}. \qquad (11)$$

The circular current $dI(r)_{circular}$ flowing in a circular strip of radius r and section $s \cdot dr$ will be, if ρ is the resistivity:

$$dI_{circular} = \frac{\mathcal{E}_{circular} \, s dr}{\rho \, 2\pi r} = \frac{\mu B}{2\pi} I_{radial} \frac{dr}{r} \qquad (12)$$

and the total circular current:

$$I_{circular} = \frac{\mu B}{2\pi} I_{radial} \ln \frac{r_2}{r_1}, \qquad (13)$$

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[8] Corbino, following Drude [17], used a dual theory of electrical conduction based on the assumption of two charge carriers, negative and positive.

[9] As pointed out by von Klitzing, the quantum Hall effect may be considered as an ideal (and quantized) version of the Corbino effect corresponding to the case in which the current in the disc, with an applied radial voltage, is only circular [18].

where μ is the electron mobility, r_1 and r_2 the inner and outer radius of the disc (we have used the relation $\mu = 1/\rho n e$).

The power dissipated in the disc is

$$W = (I^2 R)_{radial} + (I^2 R)_{circular} =$$
$$I_{radial}^2 R_{radial}(1 + \mu^2 B^2), \qquad (14)$$

where we have used equation (13) and the two relations:

$$R_{radial} = \frac{\rho}{2\pi s} \ln \frac{r_2}{r_1}, \qquad (15)$$

$$R_{circular} = \frac{\rho^2}{s^2} \frac{1}{R_{radial}}. \qquad (16)$$

Equation (14) shows that the phenomenon may be described as due to an increased resistance $R_{radial}(1 + \mu^2 B^2)$: this is the magneto-resistance effect. The circular induced emf is "distributed" homogenously along each circle. Each circular strip of section $s \cdot dr$ acts as a battery that produces current in its own resistance: therefore, the potential difference between two points arbitrarily chosen on a circle is zero. Hence, as it must be, each circle is an equipotential line.

The Faraday disc: again. – The discussion of Corbino disc helps in better understanding the physics of the Faraday disc. Let us consider a Faraday disc in which the circular symmetry is conserved. As shown above, the steady condition will be characterized by the flow of a radial and of a circular current. The mechanical power needed to keep the disc rotating with constant angular velocity ω is equal to the work per unit time done by the magnetic field on the rotating radial currents. Then, it will be given by

$$W = \int_0^{2\pi} \int_{r_1}^{r_2} (J_{radial} \, r \, d\alpha \, s)(B \, dr)(\omega \, r) =$$
$$I_{radial} \frac{1}{2} \omega B (r_2^2 - r_1^2), \qquad (17)$$

where the symbols are the same as those used in the previous section. The point is that the term

$$\mathcal{E} = \frac{1}{2} \omega B (r_2^2 - r_1^2) \qquad (18)$$

is the induced emf due only to the motion of the disc.

This emf is the source of the induced currents, radial and circular. Therefore, the physics of the Faraday disc with circular symmetry, may be summarized as follows:

a) the source of the induced currents is the induced emf due to the rotation of the disc;

b) the primary product of the induced emf is a radial current;

c) the drift velocity of the radial current produces in turn a circular induced emf that give rise to the circular current.

A possible experimental test. – The fact that the general law of electromagnetic induction explains the physics of Corbino disc, must be considered as a corroboration of the same general law in a domain usually considered as foreign to electromagnetic induction phenomena. In the following, we shall illustrates a possible experiment for testing different predictions by the general law and the flux rule.

Consider a copper ring covered by a superconducting material that prevents the magnetic field (and the vector potential) from entering the copper ring. In this situation, if we switch a static magnetic field on, there will be no induced emf in the copper ring according to the general law; however, the flux rule predicts an induced emf since the magnetic flux entering the area of the ring varies from zero to the steady value. I believe that the experiment outcome is easily predictable.

Maxwell and the electromagnetic induction. – Likely, the reader will now be curious about what Maxwell could have said about electromagnetic induction.

In the introductory and descriptive part of his *Treatise* dedicated to induction phenomena, after having reviewed Faraday's experimental results, Maxwell says:

"The whole of these phenomena may be summed up in one law. When the number of lines of magnetic induction which pass through the secondary circuit in the positive direction is altered, an electromotive force acts round the circuit, which is measured by the rate of decrease of the magnetic induction through the circuit" [19].

And: "Instead of speaking of the number of lines of magnetic force, we may speak of the magnetic induction through the circuit, or the surface-integral of magnetic induction extended over any surface bounded by the circuit" [20]. In formula (that Maxwell does not write)

$$\mathcal{E} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{B} \cdot \hat{n}\, \mathrm{d}S. \qquad (19)$$

This is the "flux rule". However, in the paragraph 598 entitled "General equations of electromotive intensity" Maxwell, treating the case of two interacting circuits and supposing that the "induced" circuit is moving (with respect to the laboratory), gets the following formula for the electromotive intensity (in modern notation)

$$\vec{E} = \vec{v} \times \vec{B} - \frac{\partial \vec{A}}{\partial t} - \mathrm{grad}\,\varphi. \qquad (20)$$

Maxwell's comments[10]:

"The electromotive intensity at a point has already been defined in Art. 68. It is also called the resultant electrical intensity, being the force which would be experienced by a unit of positive electricity placed at that point. We have now obtained the most general value of this quantity in the case of a body moving in a magnetic field due to a variable electric system. If the body is a conductor, the electromotive force will produce a current; if it is a dielectric, the electromotive force will produce only electric displacement. The electromotive intensity, or the force on a particle, must be carefully distinguished from the electromotive force along an arc of a curve, the latter quantity being the line-integral of the former. See Art. 69" [21].

And: "The electromotive force [. . .] depends on three circumstances. The first of these is the motion of the particle through the magnetic field. The part of the force depending on this motion is expressed by the first term on the right of the equation. It depends on the velocity of the particle transverse to the lines of magnetic induction. [. . .] The second term in eq. (20) depends on the time-variation of the magnetic field. This may be due either to the time-variation of the electric current in the primary circuit, or to motion of the primary circuit. [. . .] The last term is due to the variation of the function φ in different parts of the field" [22].

Three comments:

i) Maxwell says that the velocity which appear in eq. (20) is the "velocity of the particle". The calculation performed by Maxwell shows that the velocity we are speaking about is the velocity of an element of the induced (secondary) circuit[11].

ii) Apart from the meaning of $\vec{v}$, eq. (20) leads, when integrated over a closed circuit, to eq. (3) of our derivation (general law of electromagnetic induction). For Maxwell too, the "flux rule" is only a particular case of a more general law. However, Maxwell does not comment on this point.

iii) The fact that the flux rule, and not the general law discovered by Maxwell (properly modified for the interpretation of the velocity appearing in it), has become the 'law' of electromagnetic induction phenomena constitutes a puzzling historical problem.

Einstein and the electromagnetic induction. – In the incipit of his 1905 paper on relativity, Einstein speaks of asymmetries presented by "Maxwells electrodynamics, as usually understood at present"; these asymmetries "do not seem to be inherent in the phenomena". As an example, Einstein quotes the "electrodynamic interaction between a magnet and a conductor" and stresses that the observable phenomena depend only on the relative motion between the magnet and the circuit, whereas the "customary view draws a sharp distinction between the two cases, in which either the one or the other of these bodies is in motion."

At the end of paragraph six, in which the equations of fields transformation are deduced and commented,

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[10]Maxwell writes eq. (20) in terms of its components. Therefore, we have substituted, in the quotations, the reference to a vector when Maxwell refers to its components.

[11]As a matter of fact, Maxwell did not have a model for the current, because he did not have a model for electricity. Now, we easily write that $\vec{J} = ne\vec{v}$; Maxwell could not write anything similar. See paragraphs 68, 69 and 569 of the *Treatise*.

Einstein states (without carrying out any calculation) that "the asymmetry mentioned in the introduction...now disappears" [11].

We shall show that, by applying the general law (5), any asymmetry disappears. Let us consider a rigid filiform circuit and a magnet in relative rectilinear uniform motion along the common $x \equiv x'$ axis. In the reference frame of the magnet, the emf induced in the circuit is given by eq. (5) in which the velocity of the charge $\vec{v}_c$ is equal to the velocity $\vec{V}$ of the circuit along the positive direction of the x-axis (the contribution of the drift velocity is null, because the circuit is filiform). Since the magnetic field generated by the magnet does not depend explicitly on time, eq. (5) assumes the form

$$\mathcal{E} = zero + \oint \left[(\vec{V} \times \vec{B})_y \mathrm{d}y + (\vec{V} \times \vec{B})_z \mathrm{d}z \right]. \qquad (21)$$

In the reference frame of the circuit we have instead, by applying eq. (5) and by using the equations for coordinates and fields transformation

$$\mathcal{E}' = \oint \vec{E}' \cdot \vec{\mathrm{d}}l' + zero$$
$$= \Gamma \oint \left[(\vec{V} \times \vec{B})_y \mathrm{d}y + (\vec{V} \times \vec{B})_z \mathrm{d}z \right] = \Gamma \mathcal{E}, \qquad (22)$$

where $\Gamma = 1/\sqrt{1 - V^2/c^2}$. Of course, for $\Gamma \approx 1$, $\mathcal{E}' \approx \mathcal{E}$. The role of the magnetic component of the Lorentz force in the reference frame of the magnet is played, in the reference frame of the circuit, by the electric field arising from the transformation equations; however, in both frames we apply the same eq. (5): the description, as required by special relativity, is the same[12].

Conclusions. – The definition of the induced emf as the integral over a closed loop of the Lorentz force acting on a unit positive charge $(\vec{E} + \vec{v} \times \vec{B})$ leads immediately to a general law for electromagnetic induction phenomena. These are the product of two independent processes: time variation of the vector potential and effects of magnetic field on moving charges. The application of the general law to Corbino's disc yields the magneto-resistance effect without using microscopic models of electrical conduction. The flux of the magnetic field through an arbitrary surface that has the circuit as contour *is not the cause* of the induced emf. The flux rule must instead be considered as a calculation shortcut for predicting the value of the induced emf when the circuit is filiform. Maxwell wrote down "general equations of electromotive intensity" that,

integrated over a closed loop, yield the general law for electromagnetic induction, if the velocity appearing in them is correctly interpreted. Finally, the general law of electromagnetic induction yields the induced emf in both reference frames of a system composed by a magnet and a circuit in relative uniform motion, as required by special relativity.

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[20] Ref. [19], Chapt. 541, pp. 188–189.

[21] Ref. [19], Chapt. 598, pp. 239–240.

[22] Ref. [19], Chapt. 599, pp. 240–241.

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[12]The flux rule is incompatible with special relativity, because, as shown above, it implies an action at a distance with infinite velocity. Nevertheless, when $\Gamma \approx 1$ and the circuit is filiform (or equivalent to), it can be used as a calculation shortcut in both reference frames (magnet or circuit). However, it is a "good shortcut" only in simple cases (for instance, the moving bar); in the more general case discussed in this section, it is not.